EXERCISE SET 8.3

Practice Exercises

In Exercises 1–8, write the first five terms of each geometric sequence.

1. $a_1 = 5, \quad r = 3$
2. $a_1 = 4, \quad r = 3$
3. $a_1 = 20, \quad r = \frac{1}{2}$
4. $a_1 = 24, \quad r = \frac{1}{3}$
5. $a_n = -4a_{n-1}, \quad a_1 = 10$
6. $a_n = -3a_{n-1}, \quad a_1 = 10$
7. $a_n = -5a_{n-1}, \quad a_1 = -6$
8. $a_n = -6a_{n-1}, \quad a_1 = -2$

In Exercises 9–16, use the formula for the general term (the nth term) of a geometric sequence to find the indicated term of each sequence with the given first term, a_1, and common ratio, r.

9. Find a_8 when $a_1 = 6, r = 2$.
10. Find a_8 when $a_1 = 5, r = 3$.
11. Find a_{12} when $a_1 = 5, r = -2$.
12. Find a_{12} when $a_1 = 4, r = -2$.
13. Find a_{40} when $a_1 = 1000, r = -\frac{1}{2}$.
14. Find a_{30} when $a_1 = 8000, r = -\frac{1}{2}$.
15. Find a_8 when $a_1 = 1{,}000{,}000, r = 0.1$.
16. Find a_8 when $a_1 = 40{,}000, r = 0.1$.

In Exercises 17–24, write a formula for the general term (the nth term) of each geometric sequence. Then use the formula for a_n to find a_7, the seventh term of the sequence.

17. $3, 12, 48, 192, \ldots$
18. $3, 15, 75, 375, \ldots$
19. $18, 6, 2, \frac{2}{3}, \ldots$
20. $12, 6, 3, \frac{3}{2}, \ldots$
21. $1.5, -3, 6, -12, \ldots$
22. $5, -1, \frac{1}{5}, -\frac{1}{25}, \ldots$
23. $0.0004, -0.004, 0.04, -0.4, \ldots$
24. $0.0007, -0.007, 0.07, -0.7, \ldots$

Use the formula for the sum of the first n terms of a geometric sequence to solve Exercises 25–30.

25. Find the sum of the first 12 terms of the geometric sequence: $2, 6, 18, 54, \ldots$

26. Find the sum of the first 12 terms of the geometric sequence: $3, 6, 12, 24, \ldots$

27. Find the sum of the first 11 terms of the geometric sequence: $3, -6, 12, -24, \ldots$

28. Find the sum of the first 11 terms of the geometric sequence: $4, -12, 36, -108, \ldots$

29. Find the sum of the first 14 terms of the geometric sequence: $-\frac{3}{2}, 3, -6, 12, \ldots$

30. Find the sum of the first 14 terms of the geometric sequence: $-\frac{1}{24}, \frac{1}{12}, -\frac{1}{6}, \frac{1}{3}, \ldots$

In Exercises 31–36, find the indicated sum. Use the formula for the sum of the first n terms of a geometric sequence.

31. $\sum_{i=1}^{8} 3^i$ **32.** $\sum_{i=1}^{6} 4^i$ **33.** $\sum_{i=1}^{10} 5 \cdot 2^i$

34. $\sum_{i=1}^{7} 4(-3)^i$ **35.** $\sum_{i=1}^{6} \left(\frac{1}{2}\right)^{i+1}$ **36.** $\sum_{i=1}^{6} \left(\frac{1}{3}\right)^{i+1}$

In Exercises 37–44, find the sum of each infinite geometric series.

37. $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots$ **38.** $1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots$

39. $3 + \frac{3}{4} + \frac{3}{4^2} + \frac{3}{4^3} + \cdots$ **40.** $5 + \frac{5}{6} + \frac{5}{6^2} + \frac{5}{6^3} + \cdots$

41. $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots$ **42.** $3 - 1 + \frac{1}{3} - \frac{1}{9} + \cdots$

43. $\sum_{i=1}^{\infty} 8(-0.3)^{i-1}$ **44.** $\sum_{i=1}^{\infty} 12(-0.7)^{i-1}$

In Exercises 45–50, express each repeating decimal as a fraction in lowest terms.

45. $0.\overline{5} = \frac{5}{10} + \frac{5}{100} + \frac{5}{1000} + \frac{5}{10{,}000} + \cdots$

46. $0.\overline{1} = \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \frac{1}{10{,}000} + \cdots$

47. $0.\overline{47} = \frac{47}{100} + \frac{47}{10{,}000} + \frac{47}{1{,}000{,}000} + \cdots$

48. $0.\overline{83} = \frac{83}{100} + \frac{83}{10{,}000} + \frac{83}{1{,}000{,}000} + \cdots$

49. $0.\overline{257}$ **50.** $0.\overline{529}$

In Exercises 51–56, the general term of a sequence is given. Determine whether the sequence is arithmetic, geometric, or neither. If the sequence is arithmetic, find the common difference; if it is geometric, find the common ratio.

51. $a_n = n + 5$ **52.** $a_n = n - 3$

53. $a_n = 2^n$ **54.** $a_n = \left(\frac{1}{2}\right)^n$

55. $a_n = n^2 + 5$ **56.** $a_n = n^2 - 3$

Practice Plus

In Exercises 57–62, let

$$\{a_n\} = -5, 10, -20, 40, \ldots,$$
$$\{b_n\} = 10, -5, -20, -35, \ldots,$$

and

$$\{c_n\} = -2, 1, -\tfrac{1}{2}, \tfrac{1}{4}, \ldots.$$

57. Find $a_{10} + b_{10}$. **58.** Find $a_{11} + b_{11}$.

59. Find the difference between the sum of the first 10 terms of $\{a_n\}$ and the sum of the first 10 terms of $\{b_n\}$.

60. Find the difference between the sum of the first 11 terms of $\{a_n\}$ and the sum of the first 11 terms of $\{b_n\}$.

61. Find the product of the sum of the first 6 terms of $\{a_n\}$ and the sum of the infinite series containing all the terms of $\{c_n\}$.

62. Find the product of the sum of the first 9 terms of $\{a_n\}$ and the sum of the infinite series containing all the terms of $\{c_n\}$.

In Exercises 63–64, find a_2 and a_3 for each geometric sequence.

63. $8, a_2, a_3, 27$ **64.** $2, a_2, a_3, -54$

Application Exercises

Use the formula for the general term (the nth term) of a geometric sequence to solve Exercises 65–68.

In Exercises 65–66, suppose you save \$1 the first day of a month, \$2 the second day, \$4 the third day, and so on. That is, each day you save twice as much as you did the day before.

65. What will you put aside for savings on the fifteenth day of the month?

66. What will you put aside for savings on the thirtieth day of the month?

67. A professional baseball player signs a contract with a beginning salary of \$3,000,000 for the first year and an annual increase of 4% per year beginning in the second year. That is, beginning in year 2, the athlete's salary will be 1.04 times what it was in the previous year. What is the athlete's salary for year 7 of the contract? Round to the nearest dollar.

68. You are offered a job that pays \$30,000 for the first year with an annual increase of 5% per year beginning in the second year. That is, beginning in year 2, your salary will be 1.05 times what it was in the previous year. What can you expect to earn in your sixth year on the job?

In Exercises 69–70, you will develop geometric sequences that model the population growth for California and Texas, the two most-populated U.S. states.

69. The table shows the population of California for 2000 and 2010, with estimates given by the U.S. Census Bureau for 2001 through 2009.

Year	2000	2001	2002	2003	2004	2005
Population in millions	33.87	34.21	34.55	34.90	35.25	35.60

Year	2006	2007	2008	2009	2010
Population in millions	36.00	36.36	36.72	37.09	37.25

a. Divide the population for each year by the population in the preceding year. Round to two decimal places and show that California has a population increase that is approximately geometric.

b. Write the general term of the geometric sequence modeling California's population, in millions, n years after 1999.

c. Use your model from part (b) to project California's population, in millions, for the year 2020. Round to two decimal places.