37. $\begin{cases} x^2 + (y-2)^2 = 4 \\ x^2 - 2y = 0 \end{cases}$

38. $\begin{cases} x^2 - y^2 - 4x + 6y - 4 = 0 \\ x^2 + y^2 - 4x - 6y + 12 = 0 \end{cases}$

39. $\begin{cases} y = (x+3)^2 \\ x + 2y = -2 \end{cases}$

40. $\begin{cases} (x-1)^2 + (y+1)^2 = 5 \\ 2x - y = 3 \end{cases}$

41. $\begin{cases} x^2 + y^2 + 3y = 22 \\ 2x + y = -1 \end{cases}$

42. $\begin{cases} x - 3y = -5 \\ x^2 + y^2 - 25 = 0 \end{cases}$

In Exercises 43–46, let x represent one number and let y represent the other number. Use the given conditions to write a system of nonlinear equations. Solve the system and find the numbers.

43. The sum of two numbers is 10 and their product is 24. Find the numbers.

44. The sum of two numbers is 20 and their product is 96. Find the numbers.

45. The difference between the squares of two numbers is 3. Twice the square of the first number increased by the square of the second number is 9. Find the numbers.

46. The difference between the squares of two numbers is 5. Twice the square of the second number subtracted from three times the square of the first number is 19. Find the numbers.

Practice Plus

In Exercises 47–52, solve each system by the method of your choice.

47. $\begin{cases} 2x^2 + xy = 6 \\ x^2 + 2xy = 0 \end{cases}$

48. $\begin{cases} 4x^2 + xy = 30 \\ x^2 + 3xy = -9 \end{cases}$

49. $\begin{cases} -4x + y = 12 \\ y = x^3 + 3x^2 \end{cases}$

50. $\begin{cases} -9x + y = 45 \\ y = x^3 + 5x^2 \end{cases}$

51. $\begin{cases} \dfrac{3}{x^2} + \dfrac{1}{y^2} = 7 \\ \dfrac{5}{x^2} - \dfrac{2}{y^2} = -3 \end{cases}$

52. $\begin{cases} \dfrac{2}{x^2} + \dfrac{1}{y^2} = 11 \\ \dfrac{4}{x^2} - \dfrac{2}{y^2} = -14 \end{cases}$

In Exercises 53–54, make a rough sketch in a rectangular coordinate system of the graphs representing the equations in each system.

53. The system, whose graphs are a line with positive slope and a parabola whose equation has a positive leading coefficient, has two solutions.

54. The system, whose graphs are a line with negative slope and a parabola whose equation has a negative leading coefficient, has one solution.

Application Exercises

55. A planet's orbit follows a path described by $16x^2 + 4y^2 = 64$. A comet follows the parabolic path $y = x^2 - 4$. Where might the comet intersect the orbiting planet?

56. A system for tracking ships indicates that a ship lies on a path described by $2y^2 - x^2 = 1$. The process is repeated and the ship is found to lie on a path described by $2x^2 - y^2 = 1$. If it is known that the ship is located in the first quadrant of the coordinate system, determine its exact location.

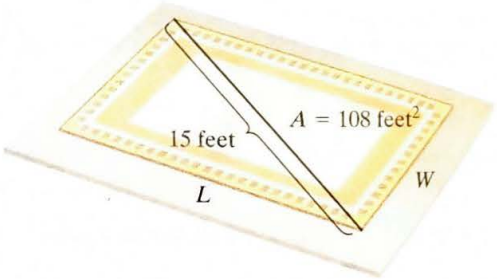
57. Find the length and width of a rectangle whose perimeter is 36 feet and whose area is 77 square feet.

58. Find the length and width of a rectangle whose perimeter is 40 feet and whose area is 96 square feet.

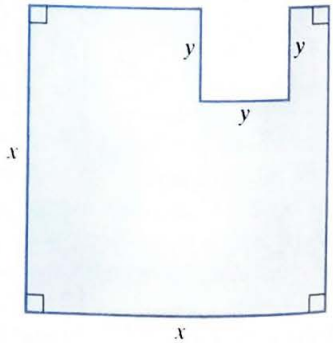
Use the formula for the area of a rectangle and the Pythagorean Theorem to solve Exercises 59–60.

59. A small television has a picture with a diagonal measure of 10 inches and a viewing area of 48 square inches. Find the length and width of the screen.

60. The area of a rug is 108 square feet and the length of its diagonal is 15 feet. Find the length and width of the rug.

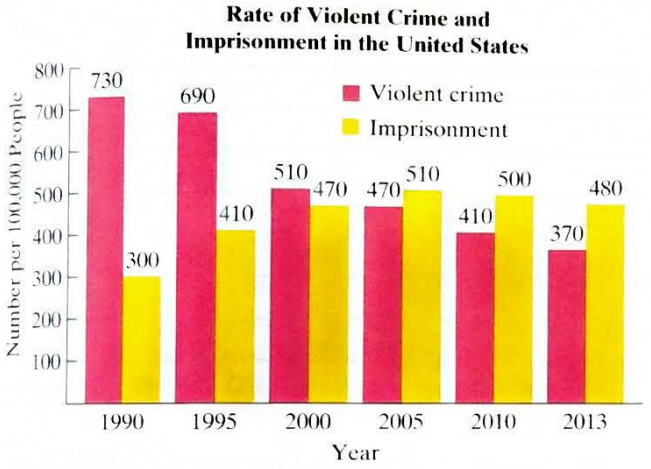


61. The figure shows a square floor plan with a smaller square area that will accommodate a combination fountain and pool. The floor with the fountain-pool area removed has an area of 21 square meters and a perimeter of 24 meters. Find the dimensions of the floor and the dimensions of the square that will accommodate the pool.



62. The area of the rectangular piece of cardboard shown below is 216 square inches. The cardboard is used to make an open box by cutting a 2-inch square from each corner and turning up the sides. If the box is to have a volume of 224 cubic inches, find the length and width of the cardboard that must be used.

63. Between 1990 and 2013, there was a drop in violent crime and a spike in the prison population in the United States. The bar graph shows the number of violent crimes per 100,000 people and the number of imprisonments per 100,000 people for six selected years from 1990 through 2013.



Sources: U.S. Justice Department; FBI

a. Based on the information in the graph, it appears that there was a year when the number of violent crimes per 100,000 Americans was the same as the number of imprisonments per 100,000 Americans. According to the graph, between which two years did this occur?

b. The data can be modeled by quadratic and linear functions.

Violent crime rate	$y = 0.6x^2 - 28x + 730$
Imprisonment rate	$-15x + y = 300$

In each function, x represents the number of years after 1990 and y represents the number per 100,000 Americans. Solve a nonlinear system to determine the year described in part (a). Round to the nearest year. How many violent crimes per 100,000 Americans and how many imprisonments per 100,000 Americans were there in that year?

Explaining the Concepts

64. What is a system of nonlinear equations? Provide an example with your description.

65. Explain how to solve a nonlinear system using the substitution method. Use $x^2 + y^2 = 9$ and $2x - y = 3$ to illustrate your explanation.

66. Explain how to solve a nonlinear system using the addition method. Use $x^2 - y^2 = 5$ and $3x^2 - 2y^2 = 19$ to illustrate your explanation.

Technology Exercises

67. Verify your solutions to any five exercises from Exercises 1–42 by using a graphing utility to graph the two equations in the system in the same viewing rectangle. Then use the intersection feature to verify the solutions.

68. Write a system of equations, one equation whose graph is a line and the other whose graph is a parabola, that has no ordered pairs that are real numbers in its solution set. Graph the equations using a graphing utility and verify that you are correct.

Critical Thinking Exercises

Make Sense? *In Exercises 69–72, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

69. I use the same steps to solve nonlinear systems as I did to solve linear systems, although I don't obtain linear equations when a variable is eliminated.

70. I graphed a nonlinear system that modeled the orbits of Earth and Mars, and the graphs indicated the system had a solution with a real ordered pair.

71. Without using any algebra, it's obvious that the nonlinear system consisting of $x^2 + y^2 = 4$ and $x^2 + y^2 = 25$ does not have real-number solutions.

72. I think that the nonlinear system consisting of $x^2 + y^2 = 36$ and $y = (x - 2)^2 - 3$ is easier to solve graphically than by using the substitution method or the addition method.

In Exercises 73–76, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

73. A system of two equations in two variables whose graphs are a circle and a line can have four real ordered-pair solutions.

74. A system of two equations in two variables whose graphs are a parabola and a circle can have four real ordered-pair solutions.

75. A system of two equations in two variables whose graphs are two circles must have at least two real ordered-pair solutions.

76. A system of two equations in two variables whose graphs are a parabola and a circle cannot have only one real ordered-pair solution.

77. The points of intersection of the graphs of $xy = 20$ and $x^2 + y^2 = 41$ are joined to form a rectangle. Find the area of the rectangle.

78. Find a and b in this figure.