Retaining the Concepts

112. Find the dimensions of a rectangle whose perimeter is 22 feet and whose area is 24 square feet. (Section 5.4, Example 5)

113. Solve using matrices. Use Gaussian elimination with back-substitution or Gauss-Jordan elimination.

$$\begin{cases} x - 2y + z = -4 \\ 2x + 2y - z = 10 \\ 4x - y + 2z = -1 \end{cases}$$

(Section 6.1, Examples 3 and 5)

114. Convert the equation

$$4x^2 + y^2 - 24x + 6y + 9 = 0$$

to standard form by completing the square on x and y. Then graph the ellipse and give the location of the foci. (Section 7.1, Example 5)

Preview Exercises

Exercises 115–117 will help you prepare for the material covered in the next section.

In Exercises 115–116, show that

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

is true for the given value of n.

115. $n = 3$: Show that $1 + 2 + 3 = \frac{3(3+1)}{2}$.

116. $n = 5$: Show that $1 + 2 + 3 + 4 + 5 = \frac{5(5+1)}{2}$.

117. Simplify: $\frac{k(k+1)(2k+1)}{6} + (k+1)^2$.

CHAPTER 8 Mid-Chapter Check Point

WHAT YOU KNOW: We learned that a sequence is a function whose domain is the set of positive integers. In an arithmetic sequence, each term after the first differs from the preceding term by a constant, the common difference, d. In a geometric sequence, each term after the first is obtained by multiplying the preceding term by a nonzero constant, the common ratio, r. We found the general term of arithmetic sequences $[a_n = a_1 + (n-1)d]$ and geometric sequences $[a_n = a_1 r^{n-1}]$ and used these formulas to find particular terms. We determined the sum of the first n terms of arithmetic sequences $\left[S_n = \frac{n}{2}(a_1 + a_n)\right]$ and geometric sequences $\left[S_n = \frac{a_1(1-r^n)}{1-r}\right]$. Finally, we determined the sum of an infinite geometric series, $a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \cdots$, if $-1 < r < 1$ $\left(S = \frac{a_1}{1-r}\right)$.

In Exercises 1–4, write the first five terms of each sequence. Assume that d represents the common difference of an arithmetic sequence and r represents the common ratio of a geometric sequence.

1. $a_n = (-1)^{n+1}\frac{n}{(n-1)!}$

2. $a_1 = 5, d = -3$

3. $a_1 = 5, r = -3$

4. $a_1 = 3, a_n = -a_{n-1} + 4$

In Exercises 5–7, write a formula for the general term (the nth term) of each sequence. Then use the formula to find the indicated term.

5. $2, 6, 10, 14, \ldots; a_{20}$

6. $3, 6, 12, 24, \ldots; a_{10}$

7. $\frac{3}{2}, 1, \frac{1}{2}, 0, \ldots; a_{30}$

8. Find the sum of the first ten terms of the sequence:

$$5, 10, 20, 40, \ldots.$$

9. Find the sum of the first 50 terms of the sequence:

$$-2, 0, 2, 4, \ldots.$$

10. Find the sum of the first ten terms of the sequence:

$$-20, 40, -80, 160, \ldots.$$

11. Find the sum of the first 100 terms of the sequence:

$$4, -2, -8, -14, \ldots.$$

In Exercises 12–15, find each indicated sum.

12. $\sum_{i=1}^{4}(i+4)(i-1)$

13. $\sum_{i=1}^{50}(3i-2)$

14. $\sum_{i=1}^{6}\left(\frac{3}{2}\right)^i$

15. $\sum_{i=1}^{\infty}\left(-\frac{2}{5}\right)^{i-1}$

16. Express $0.\overline{45}$ as a fraction in lowest terms.

17. Express the sum using summation notation. Use i for the index of summation.

$$\frac{1}{3} + \frac{2}{4} + \frac{3}{5} + \cdots + \frac{18}{20}$$

18. A skydiver falls 16 feet during the first second of a dive, 48 feet during the second second, 80 feet during the third second, 112 feet during the fourth second, and so on. Find the distance that the skydiver falls during the 15th second and the total distance the skydiver falls in 15 seconds.

19. If the average value of a house increases 10% per year, how much will a house costing \$120,000 be worth in 10 years? Round to the nearest dollar.