

In Exercises 9–12, write the system of linear equations represented by the augmented matrix. Use x , y , and z , or, if necessary, w , x , y , and z , for the variables.

$$9. \left[\begin{array}{ccc|c} 5 & 0 & 3 & -11 \\ 0 & 1 & -4 & 12 \\ 7 & 2 & 0 & 3 \end{array} \right]$$

$$10. \left[\begin{array}{ccc|c} 7 & 0 & 4 & -13 \\ 0 & 1 & -5 & 11 \\ 2 & 7 & 0 & 6 \end{array} \right]$$

$$11. \left[\begin{array}{cccc|c} 1 & 1 & 4 & 1 & 3 \\ -1 & 1 & -1 & 0 & 7 \\ 2 & 0 & 0 & 5 & 11 \\ 0 & 0 & 12 & 4 & 5 \end{array} \right]$$

$$12. \left[\begin{array}{cccc|c} 4 & 1 & 5 & 1 & 6 \\ 1 & -1 & 0 & -1 & 8 \\ 3 & 0 & 0 & 7 & 4 \\ 0 & 0 & 11 & 5 & 3 \end{array} \right]$$

In Exercises 13–18, perform each matrix row operation and write the new matrix.

$$13. \left[\begin{array}{ccc|c} 2 & -6 & 4 & 10 \\ 1 & 5 & -5 & 0 \\ 3 & 0 & 4 & 7 \end{array} \right] \frac{1}{2}R_1$$

$$14. \left[\begin{array}{ccc|c} 3 & -12 & 6 & 9 \\ 1 & -4 & 4 & 0 \\ 2 & 0 & 7 & 4 \end{array} \right] \frac{1}{3}R_1$$

$$15. \left[\begin{array}{ccc|c} 1 & -3 & 2 & 0 \\ 3 & 1 & -1 & 7 \\ 2 & -2 & 1 & 3 \end{array} \right] -3R_1 + R_2$$

$$16. \left[\begin{array}{ccc|c} 1 & -1 & 5 & -6 \\ 3 & 3 & -1 & 10 \\ 1 & 3 & 2 & 5 \end{array} \right] -3R_1 + R_2$$

$$17. \left[\begin{array}{cccc|c} 1 & -1 & 1 & 1 & 3 \\ 0 & 1 & -2 & -1 & 0 \\ 2 & 0 & 3 & 4 & 11 \\ 5 & 1 & 2 & 4 & 6 \end{array} \right] \begin{array}{l} -2R_1 + R_3 \\ -5R_1 + R_4 \end{array}$$

$$18. \left[\begin{array}{cccc|c} 1 & -5 & 2 & -2 & 4 \\ 0 & 1 & -3 & -1 & 0 \\ 3 & 0 & 2 & -1 & 6 \\ -4 & 1 & 4 & 2 & -3 \end{array} \right] \begin{array}{l} -3R_1 + R_3 \\ 4R_1 + R_4 \end{array}$$

In Exercises 19–20, a few steps in the process of simplifying the given matrix to row-echelon form, with 1s down the diagonal from upper left to lower right, and 0s below the 1s, are shown. Fill in the missing numbers in the steps that are shown.

$$19. \left[\begin{array}{ccc|c} 1 & -1 & 1 & 8 \\ 2 & 3 & -1 & -2 \\ 3 & -2 & -9 & 9 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 8 \\ 0 & 5 & \square & \square \\ 0 & 1 & \square & \square \end{array} \right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 8 \\ 0 & 1 & \square & \square \\ 0 & 1 & \square & \square \end{array} \right]$$

$$20. \left[\begin{array}{ccc|c} 1 & -2 & 3 & 4 \\ 2 & 1 & -4 & 3 \\ -3 & 4 & -1 & -2 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 3 & 4 \\ 0 & 5 & \square & \square \\ 0 & -2 & \square & \square \end{array} \right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 3 & 4 \\ 0 & 1 & \square & \square \\ 0 & -2 & \square & \square \end{array} \right]$$

In Exercises 21–38, solve each system of equations using matrices. Use Gaussian elimination with back-substitution or Gauss-Jordan elimination.

$$21. \begin{cases} x + y - z = -2 \\ 2x - y + z = 5 \\ -x + 2y + 2z = 1 \end{cases}$$

$$22. \begin{cases} x - 2y - z = 2 \\ 2x - y + z = 4 \\ -x + y - 2z = -4 \end{cases}$$

$$23. \begin{cases} x + 3y = 0 \\ x + y + z = 1 \\ 3x - y - z = 11 \end{cases}$$

$$24. \begin{cases} 3y - z = -1 \\ x + 5y - z = -4 \\ -3x + 6y + 2z = 11 \end{cases}$$

$$25. \begin{cases} 2x - y - z = 4 \\ x + y - 5z = -4 \\ x - 2y = 4 \end{cases}$$

$$26. \begin{cases} x - 3z = -2 \\ 2x + 2y + z = 4 \\ 3x + y - 2z = 5 \end{cases}$$

$$27. \begin{cases} x + y + z = 4 \\ x - y - z = 0 \\ x - y + z = 2 \end{cases}$$

$$28. \begin{cases} 3x + y - z = 0 \\ x + y + 2z = 6 \\ 2x + 2y + 3z = 10 \end{cases}$$

$$29. \begin{cases} x + 2y = z - 1 \\ x = 4 + y - z \\ x + y - 3z = -2 \end{cases}$$

$$30. \begin{cases} 2x + y = z + 1 \\ 2x = 1 + 3y - z \\ x + y + z = 4 \end{cases}$$

$$31. \begin{cases} 3a - b - 4c = 3 \\ 2a - b + 2c = -8 \\ a + 2b - 3c = 9 \end{cases}$$

$$32. \begin{cases} 3a + b - c = 0 \\ 2a + 3b - 5c = 1 \\ a - 2b + 3c = -4 \end{cases}$$

$$33. \begin{cases} 2x + 2y + 7z = -1 \\ 2x + y + 2z = 2 \\ 4x + 6y + z = 15 \end{cases}$$

$$34. \begin{cases} 3x + 2y + 3z = 3 \\ 4x - 5y + 7z = 1 \\ 2x + 3y - 2z = 6 \end{cases}$$

$$35. \begin{cases} w + x + y + z = 4 \\ 2w + x - 2y - z = 0 \\ w - 2x - y - 2z = -2 \\ 3w + 2x + y + 3z = 4 \end{cases}$$

$$36. \begin{cases} w + x + y + z = 5 \\ w + 2x - y - 2z = -1 \\ w - 3x - 3y - z = -1 \\ 2w - x + 2y - z = -2 \end{cases}$$

$$37. \begin{cases} 3w - 4x + y + z = 9 \\ w + x - y - z = 0 \\ 2w + x + 4y - 2z = 3 \\ -w + 2x + y - 3z = 3 \end{cases}$$

$$38. \begin{cases} 2w + y - 3z = 8 \\ w - x + 4z = -10 \\ 3w + 5x - y - z = 20 \\ w + x - y - z = 6 \end{cases}$$

Practice Plus

39. Find the quadratic function $f(x) = ax^2 + bx + c$ for which $f(-2) = -4$, $f(1) = 2$, and $f(2) = 0$.

40. Find the quadratic function $f(x) = ax^2 + bx + c$ for which $f(-1) = 5$, $f(1) = 3$, and $f(2) = 5$.

41. Find the cubic function $f(x) = ax^3 + bx^2 + cx + d$ for which $f(-1) = 0$, $f(1) = 2$, $f(2) = 3$, and $f(3) = 12$.

42. Find the cubic function $f(x) = ax^3 + bx^2 + cx + d$ for which $f(-1) = 3$, $f(1) = 1$, $f(2) = 6$, and $f(3) = 7$.