EXERCISE SET 3.5

Practice Exercises

In Exercises 1–8, find the domain of each rational function.

1. $f(x) = \dfrac{5x}{x - 4}$

2. $f(x) = \dfrac{7x}{x - 8}$

3. $g(x) = \dfrac{3x^2}{(x - 5)(x + 4)}$

4. $g(x) = \dfrac{2x^2}{(x - 2)(x + 6)}$

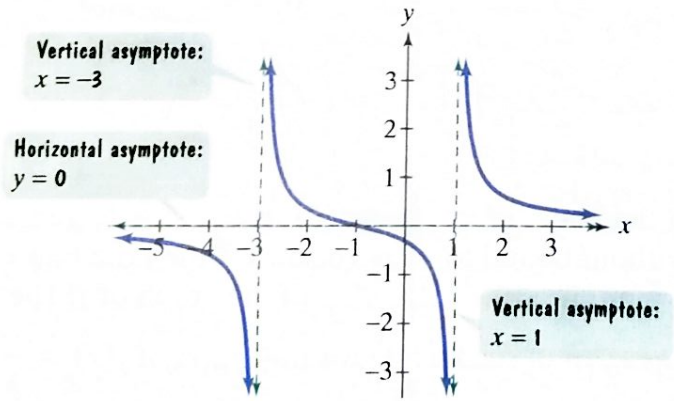
5. $h(x) = \dfrac{x + 7}{x^2 - 49}$

6. $h(x) = \dfrac{x + 8}{x^2 - 64}$

7. $f(x) = \dfrac{x + 7}{x^2 + 49}$

8. $f(x) = \dfrac{x + 8}{x^2 + 64}$

Use the graph of the rational function in the figure shown to complete each statement in Exercises 9–14.



9. As $x \to -3^-$, $f(x) \to$ _____.

10. As $x \to -3^+$, $f(x) \to$ _____.

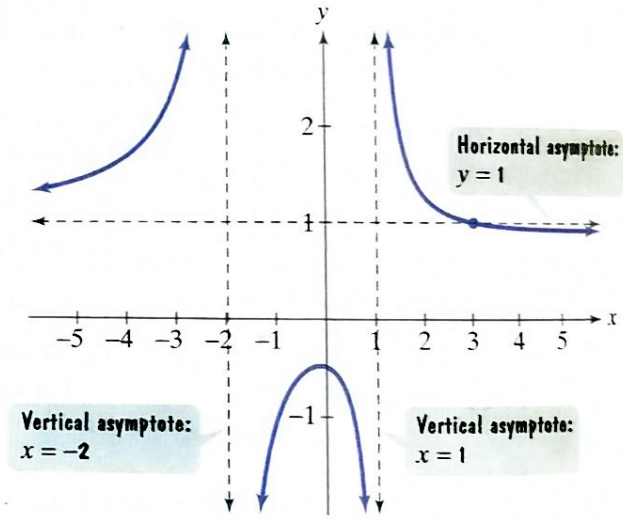
11. As $x \to 1^-$, $f(x) \to$ _____.

12. As $x \to 1^+$, $f(x) \to$ _____.

13. As $x \to -\infty$, $f(x) \to$ _____.

14. As $x \to \infty$, $f(x) \to$ _____.

Use the graph of the rational function in the figure shown to complete each statement in Exercises 15–20.



15. As $x \to 1^+$, $f(x) \to$ _____.

16. As $x \to 1^-$, $f(x) \to$ _____.

17. As $x \to -2^+$, $f(x) \to$ _____.

18. As $x \to -2^-$, $f(x) \to$ _____.

19. As $x \to \infty$, $f(x) \to$ _____.

20. As $x \to -\infty$, $f(x) \to$ _____.

In Exercises 21–36, find the vertical asymptotes, if any, and the values of x corresponding to holes, if any, of the graph of each rational function.

21. $f(x) = \dfrac{x}{x + 4}$

22. $f(x) = \dfrac{x}{x - 3}$

23. $g(x) = \dfrac{x + 3}{x(x + 4)}$

24. $g(x) = \dfrac{x + 3}{x(x - 3)}$

25. $h(x) = \dfrac{x}{x(x + 4)}$

26. $h(x) = \dfrac{x}{x(x - 3)}$

27. $r(x) = \dfrac{x}{x^2 + 4}$ **28.** $r(x) = \dfrac{x}{x^2 + 3}$

29. $f(x) = \dfrac{x^2 - 9}{x - 3}$ **30.** $f(x) = \dfrac{x^2 - 25}{x - 5}$

31. $g(x) = \dfrac{x - 3}{x^2 - 9}$ **32.** $g(x) = \dfrac{x - 5}{x^2 - 25}$

33. $h(x) = \dfrac{x + 7}{x^2 + 4x - 21}$ **34.** $h(x) = \dfrac{x + 6}{x^2 + 2x - 24}$

35. $r(x) = \dfrac{x^2 + 4x - 21}{x + 7}$ **36.** $r(x) = \dfrac{x^2 + 2x - 24}{x + 6}$

In Exercises 37–44, find the horizontal asymptote, if there is one, of the graph of each rational function.

37. $f(x) = \dfrac{12x}{3x^2 + 1}$ **38.** $f(x) = \dfrac{15x}{3x^2 + 1}$

39. $g(x) = \dfrac{12x^2}{3x^2 + 1}$ **40.** $g(x) = \dfrac{15x^2}{3x^2 + 1}$

41. $h(x) = \dfrac{12x^3}{3x^2 + 1}$ **42.** $h(x) = \dfrac{15x^3}{3x^2 + 1}$

43. $f(x) = \dfrac{-2x + 1}{3x + 5}$ **44.** $f(x) = \dfrac{-3x + 7}{5x - 2}$

In Exercises 45–56, use transformations of $f(x) = \dfrac{1}{x}$ *or* $f(x) = \dfrac{1}{x^2}$ *to graph each rational function.*

45. $g(x) = \dfrac{1}{x - 1}$ **46.** $g(x) = \dfrac{1}{x - 2}$

47. $h(x) = \dfrac{1}{x} + 2$ **48.** $h(x) = \dfrac{1}{x} + 1$

49. $g(x) = \dfrac{1}{x + 1} - 2$ **50.** $g(x) = \dfrac{1}{x + 2} - 2$

51. $g(x) = \dfrac{1}{(x + 2)^2}$ **52.** $g(x) = \dfrac{1}{(x + 1)^2}$

53. $h(x) = \dfrac{1}{x^2} - 4$ **54.** $h(x) = \dfrac{1}{x^2} - 3$

55. $h(x) = \dfrac{1}{(x - 3)^2} + 1$ **56.** $h(x) = \dfrac{1}{(x - 3)^2} + 2$

In Exercises 57–80, follow the seven steps on page 399 to graph each rational function.

57. $f(x) = \dfrac{4x}{x - 2}$ **58.** $f(x) = \dfrac{3x}{x - 1}$

59. $f(x) = \dfrac{2x}{x^2 - 4}$ **60.** $f(x) = \dfrac{4x}{x^2 - 1}$

61. $f(x) = \dfrac{2x^2}{x^2 - 1}$ **62.** $f(x) = \dfrac{4x^2}{x^2 - 9}$

63. $f(x) = \dfrac{-x}{x + 1}$ **64.** $f(x) = \dfrac{-3x}{x + 2}$

65. $f(x) = -\dfrac{1}{x^2 - 4}$ **66.** $f(x) = -\dfrac{2}{x^2 - 1}$

67. $f(x) = \dfrac{2}{x^2 + x - 2}$ **68.** $f(x) = \dfrac{-2}{x^2 - x - 2}$

69. $f(x) = \dfrac{2x^2}{x^2 + 4}$ **70.** $f(x) = \dfrac{4x^2}{x^2 + 1}$

71. $f(x) = \dfrac{x + 2}{x^2 + x - 6}$ **72.** $f(x) = \dfrac{x - 4}{x^2 - x - 6}$

73. $f(x) = \dfrac{x - 2}{x^2 - 4}$ **74.** $f(x) = \dfrac{x - 3}{x^2 - 9}$

75. $f(x) = \dfrac{x^4}{x^2 + 2}$ **76.** $f(x) = \dfrac{2x^4}{x^2 + 1}$

77. $f(x) = \dfrac{x^2 + x - 12}{x^2 - 4}$ **78.** $f(x) = \dfrac{x^2}{x^2 + x - 6}$

79. $f(x) = \dfrac{3x^2 + x - 4}{2x^2 - 5x}$ **80.** $f(x) = \dfrac{x^2 - 4x + 3}{(x + 1)^2}$

In Exercises 81–88, **a.** *Find the slant asymptote of the graph of each rational function and* **b.** *Follow the seven-step strategy and use the slant asymptote to graph each rational function.*

81. $f(x) = \dfrac{x^2 - 1}{x}$ **82.** $f(x) = \dfrac{x^2 - 4}{x}$

83. $f(x) = \dfrac{x^2 + 1}{x}$ **84.** $f(x) = \dfrac{x^2 + 4}{x}$

85. $f(x) = \dfrac{x^2 + x - 6}{x - 3}$ **86.** $f(x) = \dfrac{x^2 - x + 1}{x - 1}$

87. $f(x) = \dfrac{x^3 + 1}{x^2 + 2x}$ **88.** $f(x) = \dfrac{x^3 - 1}{x^2 - 9}$

Practice Plus

In Exercises 89–94, the equation for f is given by the simplified expression that results after performing the indicated operation. Write the equation for f and then graph the function.

89. $\dfrac{5x^2}{x^2 - 4} \cdot \dfrac{x^2 + 4x + 4}{10x^3}$ **90.** $\dfrac{x - 5}{10x - 2} \div \dfrac{x^2 - 10x + 25}{25x^2 - 1}$

91. $\dfrac{x}{2x + 6} - \dfrac{9}{x^2 - 9}$ **92.** $\dfrac{2}{x^2 + 3x + 2} - \dfrac{4}{x^2 + 4x + 3}$

93. $\dfrac{1 - \dfrac{3}{x + 2}}{1 + \dfrac{1}{x - 2}}$ **94.** $\dfrac{x - \dfrac{1}{x}}{x + \dfrac{1}{x}}$

In Exercises 95–98, use long division to rewrite the equation for g in the form

$$\text{quotient} + \frac{\text{remainder}}{\text{divisor}}.$$

Then use this form of the function's equation and transformations of $f(x) = \dfrac{1}{x}$ *to graph g.*

95. $g(x) = \dfrac{2x + 7}{x + 3}$ **96.** $g(x) = \dfrac{3x + 7}{x + 2}$

97. $g(x) = \dfrac{3x - 7}{x - 2}$ **98.** $g(x) = \dfrac{2x - 9}{x - 4}$

Application Exercises

99. A company is planning to manufacture mountain bikes. The fixed monthly cost will be \$100,000 and it will cost \$100 to produce each bicycle.

a. Write the cost function, C, of producing x mountain bikes.

b. Write the average cost function, $\overline{C}$, of producing x mountain bikes.

c. Find and interpret $\overline{C}(500)$, $\overline{C}(1000)$, $\overline{C}(2000)$, and $\overline{C}(4000)$.

d. What is the horizontal asymptote for the graph of the average cost function, $\overline{C}$? Describe what this means in practical terms.