6. True or false: Matrices of different orders can be added. ______

7. True or false: The scalar multiple $-4A$ is obtained by multiplying each element of A by -4. ______

8. If A is an $m \times n$ matrix and B is an $n \times p$ matrix, then AB is defined as an ____ $\times$ ____ matrix. To find the product AB, the number of ______ in matrix A must equal the number of ______ in matrix B.

9. True or false: Matrices of different orders can sometimes be multiplied. ______

10. True or false: Matrix multiplication is commutative. ______

EXERCISE SET 6.3

Practice Exercises

In Exercises 1–4,

a. Give the order of each matrix.

b. If $A = [a_{ij}]$, identify a_{32} and a_{23}, or explain why identification is not possible.

1. $\begin{bmatrix} 4 & -7 & 5 \\ -6 & 8 & -1 \end{bmatrix}$

2. $\begin{bmatrix} -6 & 4 & -1 \\ -9 & 0 & \frac{1}{2} \end{bmatrix}$

3. $\begin{bmatrix} 1 & -5 & \pi & e \\ 0 & 7 & -6 & -\pi \\ -2 & \frac{1}{2} & 11 & -\frac{1}{5} \end{bmatrix}$

4. $\begin{bmatrix} -4 & 1 & 3 & -5 \\ 2 & -1 & \pi & 0 \\ 1 & 0 & -e & \frac{1}{5} \end{bmatrix}$

In Exercises 5–8, find values for the variables so that the matrices in each exercise are equal.

5. $\begin{bmatrix} x \\ 4 \end{bmatrix} = \begin{bmatrix} 6 \\ y \end{bmatrix}$

6. $\begin{bmatrix} x \\ 7 \end{bmatrix} = \begin{bmatrix} 11 \\ y \end{bmatrix}$

7. $\begin{bmatrix} x & 2y \\ z & 9 \end{bmatrix} = \begin{bmatrix} 4 & 12 \\ 3 & 9 \end{bmatrix}$

8. $\begin{bmatrix} x & y+3 \\ 2z & 8 \end{bmatrix} = \begin{bmatrix} 12 & 5 \\ 6 & 8 \end{bmatrix}$

In Exercises 9–16, find the following matrices:

a. $A + B$ *b.* $A - B$

c. $-4A$ *d.* $3A + 2B$.

9. $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 9 \\ 0 & 7 \end{bmatrix}$

10. $A = \begin{bmatrix} -2 & 3 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 8 & 1 \\ 5 & 4 \end{bmatrix}$

11. $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 \\ 3 & -2 \\ 0 & 1 \end{bmatrix}$

12. $A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 2 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -3 & 6 \\ -3 & 1 & -4 \end{bmatrix}$

13. $A = \begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}$, $B = \begin{bmatrix} -5 \\ 3 \\ -1 \end{bmatrix}$

14. $A = [6 \quad 2 \quad -3]$, $B = [4 \quad -2 \quad 3]$

15. $A = \begin{bmatrix} 2 & -10 & -2 \\ 14 & 12 & 10 \\ 4 & -2 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 10 & -2 \\ 0 & -12 & -4 \\ -5 & 2 & -2 \end{bmatrix}$

16. $A = \begin{bmatrix} 6 & -3 & 5 \\ 6 & 0 & -2 \\ -4 & 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 5 & 1 \\ -1 & 2 & -6 \\ 2 & 0 & 4 \end{bmatrix}$

In Exercises 17–26, let

$$A = \begin{bmatrix} -3 & -7 \\ 2 & -9 \\ 5 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} -5 & -1 \\ 0 & 0 \\ 3 & -4 \end{bmatrix}.$$

Solve each matrix equation for X.

17. $X - A = B$

18. $X - B = A$

19. $2X + A = B$

20. $3X + A = B$

21. $3X + 2A = B$

22. $2X + 5A = B$

23. $B - X = 4A$

24. $A - X = 4B$

25. $4A + 3B = -2X$

26. $4B + 3A = -2X$

In Exercises 27–36, find (if possible) the following matrices:

a. AB *b.* BA.

27. $A = \begin{bmatrix} 1 & 3 \\ 5 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -2 \\ -1 & 6 \end{bmatrix}$

28. $A = \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 5 & -6 \end{bmatrix}$

29. $A = [1 \quad 2 \quad 3 \quad 4]$, $B = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$

30. $A = \begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix}$, $B = [1 \quad 2 \quad 3]$

31. $A = \begin{bmatrix} 1 & -1 & 4 \\ 4 & -1 & 3 \\ 2 & 0 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 4 \\ 1 & -1 & 3 \end{bmatrix}$

32. $A = \begin{bmatrix} 1 & -1 & 1 \\ 5 & 0 & -2 \\ 3 & -2 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -4 & 5 \\ 3 & -1 & 2 \end{bmatrix}$

33. $A = \begin{bmatrix} 4 & 2 \\ 6 & 1 \\ 3 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 & 4 \\ -1 & -2 & 0 \end{bmatrix}$

34. $A = \begin{bmatrix} 2 & 4 \\ 3 & 1 \\ 4 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 2 & 0 \\ -1 & -3 & 5 \end{bmatrix}$

35. $A = \begin{bmatrix} 2 & -3 & 1 & -1 \\ 1 & 1 & -2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ -1 & 1 \\ 5 & 4 \\ 10 & 5 \end{bmatrix}$

36. $A = \begin{bmatrix} 2 & -1 & 3 & 2 \\ 1 & 0 & -2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 2 \\ 1 & 1 \\ 3 & -4 \\ 6 & 5 \end{bmatrix}$

In Exercises 37–44, perform the indicated matrix operations given that A, B, and C are defined as follows. If an operation is not defined, state the reason.

$$A = \begin{bmatrix} 4 & 0 \\ -3 & 5 \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 1 \\ -2 & -2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

37. $4B - 3C$

38. $5C - 2B$

39. $BC + CB$

40. $A(B + C)$

41. $A - C$

42. $B - A$

43. $A(BC)$

44. $A(CB)$

Practice Plus

In Exercises 45–50, let

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$D = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}.$$

45. Find the product of the sum of A and B and the difference between C and D.

46. Find the product of the difference between A and B and the sum of C and D.

47. Use any three of the matrices to verify a distributive property.

48. Use any three of the matrices to verify an associative property.

In Exercises 49–50, suppose that the vertices of a computer graphic are points, (x, y), represented by the matrix

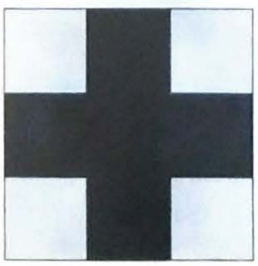
$$Z = \begin{bmatrix} x \\ y \end{bmatrix}.$$

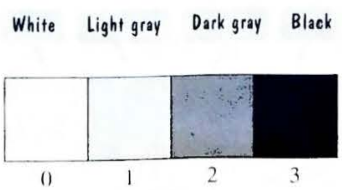
49. Find BZ and explain why this reflects the graphic about the x-axis.

50. Find CZ and explain why this reflects the graphic about the y-axis.

Application Exercises

The + sign in the figure is shown using 9 pixels in a 3×3 grid. The color levels are given to the right of the figure. Each color is represented by a specific number: 0, 1, 2, or 3. Use this information to solve Exercises 51–52.





51. a. Find a matrix that represents a digital photograph of the + sign.

b. Adjust the contrast by changing the black to dark gray and the light gray to white. Use matrix addition to accomplish this.

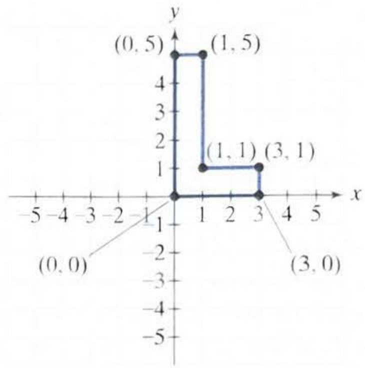
c. Adjust the contrast by changing the black to light gray and the light gray to dark gray. Use matrix addition to accomplish this.

52. a. Find a matrix that represents a digital photograph of the + sign.

b. Adjust the contrast by changing the black to dark gray and the light gray to black. Use matrix addition to accomplish this.

c. Adjust the contrast by leaving the black alone and changing the light gray to white. Use matrix addition to accomplish this.

The figure shows the letter L in a rectangular coordinate system.



The figure can be represented by the matrix

$$B = \begin{bmatrix} 0 & 3 & 3 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 5 & 5 \end{bmatrix}.$$

Each column in the matrix describes a point on the letter. The order of the columns shows the direction in which a pencil must move to draw the letter. The L is completed by connecting the last point in the matrix, (0, 5), to the starting point, (0, 0). Use these ideas to solve Exercises 53–60.

53. Use matrix operations to move the L 2 units to the left and 3 units down. Then graph the letter and its transformation in a rectangular coordinate system.

54. Use matrix operations to move the L 2 units to the right and 3 units down. Then graph the letter and its transformation in a rectangular coordinate system.

55. Reduce the L to half its perimeter and move the reduced image 1 unit up. Then graph the letter and its transformation.

56. Reduce the L to half its perimeter and move the reduced image 2 units up. Then graph the letter and its transformation.

57. a. If $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, find AB.

b. Graph the object represented by matrix AB. What effect does the matrix multiplication have on the letter L represented by matrix B?

58. a. If $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$, find AB.

b. Graph the object represented by matrix AB. What effect does the matrix multiplication have on the letter L represented by matrix B?