## CONCEPT AND VOCABULARY CHECK

*Fill in each blank so that the resulting statement is true.*

1. The product rule for logarithms states that $\log_b(MN) =$ ________. The logarithm of a product is the ______ of the logarithms.
2. The quotient rule for logarithms states that $\log_b\left(\frac{M}{N}\right) =$ ________. The logarithm of a quotient is the ________ of the logarithms.
3. The power rule for logarithms states that $\log_b M^p =$ ______. The logarithm of a number with an exponent is the ________ of the exponent and the logarithm of that number.
4. The change-of-base property for logarithms allows us to write logarithms with base $b$ in terms of a new base $a$. Introducing base $a$, the property states that

$$\log_b M = \frac{\_\_\_\_}{\_\_\_\_}.$$

## EXERCISE SET 4.3

### Practice Exercises

*In Exercises 1–40, use properties of logarithms to expand each logarithmic expression as much as possible. Where possible, evaluate logarithmic expressions without using a calculator.*

1. $\log_5(7 \cdot 3)$
2. $\log_8(13 \cdot 7)$
3. $\log_7(7x)$
4. $\log_9(9x)$
5. $\log(1000x)$
6. $\log(10{,}000x)$
7. $\log_7\left(\frac{7}{x}\right)$
8. $\log_9\left(\frac{9}{x}\right)$
9. $\log\left(\frac{x}{100}\right)$
10. $\log\left(\frac{x}{1000}\right)$
11. $\log_4\left(\frac{64}{y}\right)$
12. $\log_5\left(\frac{125}{y}\right)$
13. $\ln\left(\frac{e^2}{5}\right)$
14. $\ln\left(\frac{e^4}{8}\right)$
15. $\log_b x^3$
16. $\log_b x^7$
17. $\log N^{-6}$
18. $\log M^{-8}$
19. $\ln \sqrt[5]{x}$
20. $\ln \sqrt[7]{x}$
21. $\log_b(x^2y)$
22. $\log_b(xy^3)$
23. $\log_4\left(\frac{\sqrt{x}}{64}\right)$
24. $\log_5\left(\frac{\sqrt{x}}{25}\right)$
25. $\log_6\left(\frac{36}{\sqrt{x+1}}\right)$
26. $\log_8\left(\frac{64}{\sqrt{x+1}}\right)$
27. $\log_b\left(\frac{x^2y}{z^2}\right)$
28. $\log_b\left(\frac{x^3y}{z^2}\right)$
29. $\log \sqrt{100x}$
30. $\ln \sqrt{ex}$
31. $\log \sqrt[3]{\frac{x}{y}}$
32. $\log \sqrt[5]{\frac{x}{y}}$
33. $\log_b\left(\frac{\sqrt{x}y^3}{z^3}\right)$
34. $\log_b\left(\frac{\sqrt[3]{xy^4}}{z^5}\right)$
35. $\log_5 \sqrt[3]{\frac{x^2y}{25}}$
36. $\log_2 \sqrt[5]{\frac{xy^4}{16}}$
37. $\ln\left[\frac{x^3\sqrt{x^2+1}}{(x+1)^4}\right]$
38. $\ln\left[\frac{x^4\sqrt{x^2+3}}{(x+3)^5}\right]$
39. $\log\left[\frac{10x^2\sqrt[3]{1-x}}{7(x+1)^2}\right]$
40. $\log\left[\frac{100x^3\sqrt[3]{5-x}}{3(x+7)^2}\right]$

*In Exercises 41–70, use properties of logarithms to condense each logarithmic expression. Write the expression as a single logarithm whose coefficient is 1. Where possible, evaluate logarithmic expressions without using a calculator.*

41. $\log 5 + \log 2$
42. $\log 250 + \log 4$
43. $\ln x + \ln 7$
44. $\ln x + \ln 3$
45. $\log_2 96 - \log_2 3$
46. $\log_3 405 - \log_3 5$
47. $\log(2x + 5) - \log x$
48. $\log(3x + 7) - \log x$
49. $\log x + 3 \log y$
50. $\log x + 7 \log y$
51. $\frac{1}{2}\ln x + \ln y$
52. $\frac{1}{3}\ln x + \ln y$
53. $2 \log_b x + 3 \log_b y$
54. $5 \log_b x + 6 \log_b y$
55. $5 \ln x - 2 \ln y$
56. $7 \ln x - 3 \ln y$
57. $3 \ln x - \frac{1}{3}\ln y$
58. $2 \ln x - \frac{1}{2}\ln y$
59. $4 \ln(x + 6) - 3 \ln x$
60. $8 \ln(x + 9) - 4 \ln x$
61. $3 \ln x + 5 \ln y - 6 \ln z$
62. $4 \ln x + 7 \ln y - 3 \ln z$
63. $\frac{1}{2}(\log x + \log y)$
64. $\frac{1}{3}(\log_4 x - \log_4 y)$
65. $\frac{1}{2}(\log_5 x + \log_5 y) - 2 \log_5(x + 1)$
66. $\frac{1}{3}(\log_4 x - \log_4 y) + 2 \log_4(x + 1)$
67. $\frac{1}{3}[2 \ln(x + 5) - \ln x - \ln(x^2 - 4)]$
68. $\frac{1}{3}[5 \ln(x + 6) - \ln x - \ln(x^2 - 25)]$
69. $\log x + \log(x^2 - 1) - \log 7 - \log(x + 1)$
70. $\log x + \log(x^2 - 4) - \log 15 - \log(x + 2)$

*In Exercises 71–78, use common logarithms or natural logarithms and a calculator to evaluate to four decimal places.*

71. $\log_5 13$
72. $\log_6 17$
73. $\log_{14} 87.5$
74. $\log_{16} 57.2$
75. $\log_{0.1} 17$
76. $\log_{0.3} 19$
77. $\log_\pi 63$
78. $\log_\pi 400$

*In Exercises 79–82, use a graphing utility and the change-of-base property to graph each function.*

79. $y = \log_3 x$
80. $y = \log_{15} x$
81. $y = \log_2(x + 2)$
82. $y = \log_3(x - 2)$

### Practice Plus

*In Exercises 83–88, let $\log_b 2 = A$ and $\log_b 3 = C$. Write each expression in terms of $A$ and $C$.*

83. $\log_b \frac{3}{2}$
84. $\log_b 6$
85. $\log_b 8$
86. $\log_b 81$
87. $\log_b \sqrt{\frac{2}{27}}$
88. $\log_b \sqrt{\frac{3}{16}}$