6.1 Properties of Exponents *(pp. 291–298)*

Simplify $\left(\dfrac{x}{4}\right)^{-4}$. Write your answer using only positive exponents.

$$\left(\frac{x}{4}\right)^{-4} = \frac{x^{-4}}{4^{-4}}$$ Power of a Quotient Property

$$= \frac{4^4}{x^4}$$ Definition of negative exponent

$$= \frac{256}{x^4}$$ Simplify.

Simplify the expression. Write your answer using only positive exponents.

1. $y^3 \cdot y^{-5}$ **2.** $\dfrac{x^4}{x^7}$ **3.** $(x^0 y^2)^3$ **4.** $\left(\dfrac{2x^2}{5y^4}\right)^{-2}$

6.2 Radicals and Rational Exponents *(pp. 299–304)*

Evaluate $512^{1/3}$.

$$512^{1/3} = \sqrt[3]{512}$$ Rewrite the expression in radical form.

$$= \sqrt[3]{8 \cdot 8 \cdot 8}$$ Rewrite the expression showing factors.

$$= 8$$ Evaluate the cube root.

Evaluate the expression.

5. $\sqrt[3]{8}$ **6.** $\sqrt[5]{-243}$ **7.** $625^{3/4}$ **8.** $(-25)^{1/2}$

6.3 Exponential Functions *(pp. 305–312)*

Graph $f(x) = 9(3)^x$.

Step 1 Make a table of values.

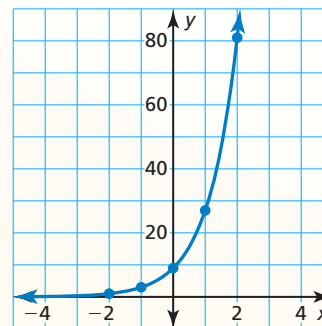| x	−2	−1	0	1	2
f(x)	1	3	9	27	81

Step 2 Plot the ordered pairs.

Step 3 Draw a smooth curve through the points.

Graph the function. Describe the domain and range.

9. $f(x) = -4\left(\dfrac{1}{4}\right)^x$ **10.** $f(x) = 3^{x+2}$ **11.** $f(x) = 2^{x-4} - 3$

12. Write and graph an exponential function f represented by the table. Then compare the graph to the graph of $g(x) = \left(\dfrac{1}{2}\right)^x$.

x	0	1	2	3
y	2	1	0.5	0.25

6.4 Exponential Growth and Decay *(pp. 313–322)*

Rewrite the function $y = 10(0.65)^{t/8}$ to determine whether it represents *exponential growth* or *exponential decay*. Identify the percent rate of change.

$$y = 10(0.65)^{t/8} \qquad \text{Write the function.}$$
$$= 10(0.65^{1/8})^t \qquad \text{Power of a Power Property}$$
$$\approx 10(0.95)^t \qquad \text{Evaluate the power.}$$

The function is of the form $y = a(1 - r)^t$, where $1 - r < 1$, so it represents exponential decay. Use the decay factor $1 - r$ to find the rate of decay.

$$1 - r = 0.95 \qquad \text{Write an equation.}$$
$$r = 0.05 \qquad \text{Solve for } r.$$

▶ So, the function represents exponential decay, and the rate of decay is 5%.

Determine whether the table represents an *exponential growth function,* an *exponential decay function,* or *neither.* Explain.

13.

x	0	1	2	3
y	3	6	12	24

14.

x	1	2	3	4
y	162	108	72	48

Rewrite the function to determine whether it represents *exponential growth* or *exponential decay.* Identify the percent rate of change.

15. $f(t) = 4(1.25)^{t + 3}$ **16.** $y = (1.06)^{8t}$ **17.** $f(t) = 6(0.84)^{t - 4}$

18. You deposit $750 in a savings account that earns 5% annual interest compounded quarterly. (a) Write a function that represents the balance after t years. (b) What is the balance of the account after 4 years?

19. The value of a TV is $1500. Its value decreases by 14% each year. (a) Write a function that represents the value y (in dollars) of the TV after t years. (b) Find the approximate monthly percent decrease in value. (c) Graph the function from part (a). Use the graph to estimate the value of the TV after 3 years.

6.5 Solving Exponential Equations *(pp. 325–330)*

Solve $\frac{1}{9} = 3^{x + 6}$.

$$\frac{1}{9} = 3^{x + 6} \qquad \text{Write the equation.}$$
$$3^{-2} = 3^{x + 6} \qquad \text{Rewrite } \tfrac{1}{9} \text{ as } 3^{-2}.$$
$$-2 = x + 6 \qquad \text{Equate the exponents.}$$
$$x = -8 \qquad \text{Solve for } x.$$

Solve the equation.

20. $5^x = 5^{3x - 2}$ **21.** $3^{x - 2} = 1$ **22.** $-4 = 6^{4x - 3}$

23. $\left(\frac{1}{3}\right)^{2x + 3} = 5$ **24.** $\left(\frac{1}{16}\right)^{3x} = 64^{2(x + 8)}$ **25.** $27^{2x + 2} = 81^{x + 4}$

6.6 Geometric Sequences *(pp. 331–338)*

Write the next three terms of the geometric sequence 2, 6, 18, 54, ...

Use a table to organize the terms and extend the sequence.

Position	1	2	3	4	5	6	7
Term	2	6	18	54	162	486	1458

Each term is 3 times the previous term. So, the common ratio is 3.

$\times 3$ $\times 3$ $\times 3$ $\times 3$ $\times 3$ $\times 3$

Multiply a term by 3 to find the next term.

▶ The next three terms are 162, 486, and 1458.

Decide whether the sequence is *arithmetic*, *geometric*, or *neither*. Explain your reasoning. If the sequence is geometric, write the next three terms and graph the sequence.

26. 3, 12, 48, 192, . . . **27.** 9, −18, 27, −36, . . . **28.** 375, −75, 15, −3, . . .

Write an equation for the *n*th term of the geometric sequence. Then find a_9.

29. 1, 4, 16, 64, . . . **30.** 5, −10, 20, −40, . . . **31.** 486, 162, 54, 18, . . .

6.7 Recursively Defined Sequences *(pp. 339–346)*

Write a recursive rule for the sequence 5, 12, 19, 26, 33, ...

Use a table to organize the terms and find the pattern.

Position, n	1	2	3	4	5
Term, a_n	5	12	19	26	33

$+7$ $+7$ $+7$ $+7$

The sequence is arithmetic, with first term $a_1 = 5$ and common difference $d = 7$.

$a_n = a_{n-1} + d$ Recursive equation for an arithmetic sequence

$a_n = a_{n-1} + 7$ Substitute 7 for d.

▶ So, a recursive rule for the sequence is $a_1 = 5$, $a_n = a_{n-1} + 7$.

Write the first six terms of the sequence. Then graph the sequence.

32. $a_1 = 4, a_n = a_{n-1} + 5$ **33.** $a_1 = -4, a_n = -3a_{n-1}$ **34.** $a_1 = 32, a_n = \frac{1}{4}a_{n-1}$

Write a recursive rule for the sequence.

35. 3, 8, 13, 18, 23, . . . **36.** 3, 6, 12, 24, 48, . . . **37.** 7, 6, 13, 19, 32, . . .

38. The first term of a sequence is 8. Each term of the sequence is 5 times the preceding term. Graph the first four terms of the sequence. Write a recursive rule and an explicit rule for the sequence.

6 Chapter Test

Evaluate the expression.

1. $-\sqrt[4]{16}$

2. $729^{1/6}$

3. $(-32)^{7/5}$

Simplify the expression. Write your answer using only positive exponents.

4. $z^{-2} \cdot z^4$

5. $\dfrac{b^{-5}}{a^0 b^{-8}}$

6. $\left(\dfrac{2c^4}{5}\right)^{-3}$

Write and graph a function that represents the situation.

7. Your starting annual salary of $42,500 increases by 3% each year.

8. You deposit $500 in an account that earns 6.5% annual interest compounded yearly.

Write an explicit rule and a recursive rule for the sequence.

9.

n	1	2	3	4
a_n	-6	8	22	36

10.

n	1	2	3	4
a_n	400	100	25	6.25

Solve the equation. Check your solution.

11. $2^x = \dfrac{1}{128}$

12. $256^{x+2} = 16^{3x-1}$

13. Graph $f(x) = 2(6)^x$. Compare the graph to the graph of $g(x) = 6^x$. Describe the domain and range of f.

Use the equation to complete the statement "$a \;\boxed{}\; b$" with the symbol $<$, $>$, or $=$. Do not attempt to solve the equation.

14. $\dfrac{5^a}{5^b} = 5^{-3}$

15. $9^a \cdot 9^{-b} = 1$

16. The first two terms of a sequence are $a_1 = 3$ and $a_2 = -12$. Let a_3 be the third term when the sequence is arithmetic and let b_3 be the third term when the sequence is geometric. Find $a_3 - b_3$.

17. At sea level, Earth's atmosphere exerts a pressure of 1 atmosphere. Atmospheric pressure P (in atmospheres) decreases with altitude. It can be modeled by $P = (0.99988)^a$, where a is the altitude (in meters).

    a. Identify the initial amount, decay factor, and decay rate.

    b. Use a graphing calculator to graph the function. Use the graph to estimate the atmospheric pressure at an altitude of 5000 feet.

18. You follow the training schedule from your coach.

    a. Write an explicit rule and a recursive rule for the geometric sequence.

    b. On what day do you run approximately 3 kilometers?

> **Training On Your Own**
> **Day 1:** Run 1 km.
> **Each day after Day 1:** Run 20% farther than the previous day.