Chapter 6

Name____ Date__________

Chapter 6 Exponential Functions and Sequences

Dear Family,

In this chapter, your child will be learning more about exponents and their use in the real world. What is an exponent? Exponents are used to show repeated multiplication. Anything that grows or shrinks at a steady rate (percent of change)—population, the viewership of online videos, or a certificate of deposit—can be described with exponents.

For example, suppose that the population of deer doubles every year in a remote area. Then in 1 year, there would be twice (2^1) as many deer, after 2 years, there would be four (2^2) times as many deer, and after 3 years, there would be eight (2^3) times as many deer.

- After 5 years, if none of the deer were hunted, how many deer could be found in this remote area? $(2^5 = 32 \text{ times as many})$
- After 10 years, how many deer could be found in this remote area?

Consider the Rule of 70 for exponential growth. This rule is useful in finding the doubling time of a quantity growing at a given percentage rate. To find the doubling time, divide the percentage number into 70. This will give you the number of years required to double. For example, at a 10% growth rate, doubling time would be $\frac{70}{10} = 7$ years.

To find the annual growth rate, divide 70 by the doubling time. For example, $\frac{70}{21} \approx 3$, or a 3% annual growth rate.

- At a 7% growth rate, what would be the doubling time?
- What would the approximate annual growth rate (rounded to the nearest whole number) be after 12 years?
- Using a source such as a newspaper or the Internet, find the yearly rate (percent) of growth for something that interests your student. Figure out how many years it will take for the initial amount to double at this rate. Do you think the growth is sustainable? If not, why not? What factors could affect the growth at this rate?

How does the Rule of 70 work? Use the Internet to research the math behind the rule.

Nombre ____ Fecha ________

Capítulo 6 Funciones exponenciales y secuencias

Estimada familia:

En este capítulo, su hijo aprenderá más sobre exponentes y su uso en el mundo real. ¿Qué es un exponente? Los exponentes se usan para mostrar la multiplicación repetida. Cualquier cosa que aumente o disminuya a una tasa constante (porcentaje de cambio) —una población, la audiencia de videos en línea o un certificado de depósito— puede describirse con exponentes.

Por ejemplo, supongan que la población de ciervos se duplica todos los años en un área remota. Luego, en un 1 año, habrá el doble $\left(2^1\right)$ de ciervos y después de 2 años, habrá el cuádruple $\left(2^2\right)$ de ciervos, y después de 3 años, habrá ocho $\left(2^3\right)$ veces más ciervos.

- Después 5 años, si no se cazó a un ningún ciervo, ¿cuántos ciervos podrían hallarse en esta área remota? $\left(2^5 = 32 \text{ veces más ciervos}\right)$
- Después de 10 años, ¿cuántos ciervos podrían hallarse en esta área remota?

Consideren la regla del 70 para el crecimiento exponencial. Esta regla es útil para hallar el tiempo de duplicación de una cantidad que crece a una tasa de porcentaje dada. Para hallar el tiempo de duplicación, dividan el número de porcentaje en 70. Esto les dará el número de años necesarios para duplicarse. Por ejemplo, a una tasa de crecimiento del 10% , el tiempo de duplicación sería $\frac{70}{10} = 7$ años.

Para hallar la tasa de crecimiento anual, dividan 70 entre el tiempo de duplicación. Por ejemplo, $\frac{70}{21} \approx 3$, o una tasa de crecimiento anual del 3%.

- A una tasa de crecimiento del 7%, ¿cuál sería el tiempo de duplicación?
- ¿Cuál sería la tasa de crecimiento anual aproximada (redondeada al número entero más cercano) después de 12 años?
- Usen una fuente tal como un periódico o Internet para hallar la tasa anual (porcentaje) de crecimiento de algo que le interese a su hijo. Calculen cuántos años tardará la cantidad inicial en duplicarse a esta tasa. ¿Creen que este crecimiento es sustentable? Si no lo creen, ¿por qué no? ¿Qué factores podrían afectar el crecimiento a esta tasa?

¿Cómo funciona la regla del 70? Consulten en Internet para investigar sobre las matemáticas detrás de la regla.

6.1 Start Thinking

The result of basic operations with variables may be determined using several numbers and observing patterns. Choose any two numbers greater than 1 and designate the first to be the base and the other to be the power. (In the expression x^n, x is the base and n is the power.) Complete the table by performing each operation at least twice, using different numbers each time and describing any patterns you notice.

Operation	Example 1	Example 2	Pattern
$x^n + x^n$			
$x^n \bullet x^n$			
$\frac{x^n}{x^n}$			

6.1 Warm Up

Simplify.

1. 7^1

2. $(-1)^2$

3. $(1.2)^3$

4. $(1.4)^3 \bullet 2^4$

5. $(-3)(-1)$

6. $(-1) \bullet 1^4 \bullet (-4)^3$

6.1 Cumulative Review Warm Up

Graph the linear equation.

1. $y = 3$

2. $x = -2$

3. $x - 4y = 12$

4. $-3x - 7y = -14$

5. $2x + 7y = 1$

6. $6x - 8y = 32$

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6.1 Practice A

In Exercises 1–6, evaluate the expression.

1. $(-3)^0$

2. 7^0

3. 3^{-5}

4. $(-5)^{-3}$

5. $\dfrac{3^{-2}}{9^0}$

6. $\dfrac{6^{-1}}{-5^0}$

In Exercises 7–18, simplify the expression. Write your answer using only positive exponents.

7. x^{-6}

8. z^0

9. $7x^{-4}y^0$

10. $12f^0g^{-9}$

11. $\dfrac{3^{-2}a^0}{b^{-2}}$

12. $\dfrac{6^0tu^{-5}}{2^5}$

13. $\dfrac{4^7}{4^4}$

14. $\dfrac{(-3)^6}{(-3)^3}$

15. $(-8)^3 \bullet (-8)^3$

16. $7^{-4} \bullet 7^4$

17. $(h^3)^4$

18. $(t^{-2})^6$

19. A camera lens magnifies an object 10^3 times. The length of an object is 10^{-4} centimeter. What is its magnified length?

In Exercises 20–22, simplify the expression. Write your answer using only positive exponents.

20. $(-2y)^5$

21. $(3d)^{-3}$

22. $\left(\dfrac{5}{b}\right)^{-3}$

In Exercises 23 and 24, simplify the expression. Write your answer using only positive exponents.

23. $\left(\dfrac{3x^2y^{-3}}{2x^{-3}y^2}\right)^3$

24. $\left(\dfrac{-6a^{-9}b^5}{2a^2b^{-4}}\right)^4$

In Exercises 25 and 26, evaluate the expression. Write your answer in scientific notation and standard form.

25. $(1.2 \times 10^7)(4 \times 10^{-2})$

26. $\dfrac{3.9 \times 10^8}{1.3 \times 10^3}$

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6.1 Practice B

In Exercises 1–6, evaluate the expression.

1. 5^{-4}

2. $(-5)^{-4}$

3. $\dfrac{7^{-1}}{-8^{0}}$

4. $\dfrac{8^{-1}}{(-4)^{0}}$

5. $\dfrac{-2^{-4}}{3^{-3}}$

6. $\dfrac{6^{-2}}{(-1)^{-4}}$

In Exercises 7–21, simplify the expression. Write your answer using only positive exponents.

7. $\dfrac{7^{-2}m^{0}}{n^{-4}}$

8. $\dfrac{(-9)^{0}j^{-1}k^{-4}}{2^{0}}$

9. $\dfrac{5^{-2}w^{0}}{y^{-10}}$

10. $\dfrac{t^{-5}}{8^{-2}s^{-3}}$

11. $\dfrac{3^{-2}a^{-1}}{9^{-1}b^{-2}c^{0}}$

12. $\dfrac{17x^{0}y^{-8}}{4^{-2}z^{-6}}$

13. $(p^{6})^{3}$

14. $(q^{-4})^{5}$

15. $5^{3} \bullet 5^{-7}$

16. $-4 \bullet (-4)^{-2}$

17. $\dfrac{x^{7}}{x^{4}} \bullet x^{2}$

18. $\dfrac{v^{5} \bullet v^{3}}{v^{2}}$

19. $(-8t^{2})^{3}$

20. $\left(-\dfrac{q^{4}}{5}\right)^{-3}$

21. $\left(\dfrac{1}{3h^{5}}\right)^{-4}$

In Exercises 22 and 23, simplify the expression. Write your answer using only positive exponents.

22. $\left(\dfrac{5x^{-4}y^{3}}{2x^{2}y^{0}}\right)^{2} \bullet \left(\dfrac{4xy}{y^{3}}\right)^{2}$

23. $\left(\dfrac{2a^{0}b^{-4}}{b^{3}}\right)^{4} \bullet \left(\dfrac{a^{3}b^{-2}}{3b^{4}a^{-4}}\right)^{3}$

In Exercises 24 and 25, evaluate the expression. Write your answer in scientific notation and standard form.

24. $(4.3 \times 10^{-4})(6 \times 10^{7})$

25. $\dfrac{1.2 \times 10^{-3}}{4.8 \times 10^{-10}}$

26. Find x and y when $b^{x}b^{y} = b^{8}$ and $b^{4x}b^{-2y} = b^{2}$. Explain how you found your answer.

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6.1 Enrichment and Extension

Properties of Exponents

Simplify the expression. Write your answer using only positive exponents.

1. $\left(\frac{-3mn^2p^{-6}}{4mn}\right) \bullet \left(\frac{9m^{-2}p^2}{16mn^4p^5}\right)^{-1}$

2. $xy^3z^{-4} \bullet x^{-5} \bullet xz^{-4}y^{-3} \bullet x^0z$

3. $\frac{-4\left(-3xy^{-3}z^4\right)^{-2}}{2x^{-5}yz^5}$

4. $\frac{\frac{-4mp^{-3}q^2}{25m^2p^4}}{\frac{16mpq^2}{15m^2p^{-2}}}$

5. $x^2y^3z^{-4} \bullet \frac{x^5yz^{-7}}{x^{-1}y^{-1}z^4}$

6. $\left(\left(x^{-3}y\right)^{-2}\right)^3 \bullet \left(x^4y^{-2}\right)^{-1}$

7. $\left(\left(\frac{-2ab^2c}{4a^{-5}b^{-3}c^6}\right)^3\right)^{-2}$

8. $\left(\frac{\left(\frac{zx^2y^{-4}}{xy^5x^{-2}}\right)^{-2} \bullet \frac{-2}{x^{-6}y^7}}{\frac{4xyz^{-3}}{x^{-6}}}\right)^{-3}$

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6.1 Puzzle Time

Why Do Bees Have Sticky Hair?

Write the letter of each answer in the box containing the exercise number.

Simplify the expression. Write your answer using only positive exponents, when necessary.

1. $(-9)^0$

2. 4^{-2}

3. $(-6)^{-3}$

4. $\dfrac{8^{-3}}{4^0}$

5. $\dfrac{12^{-2}}{(-13)^0}$

6. $\dfrac{(9^{-2})}{(3^{-3})}$

7. $15x^0y^{-2}$

8. $21x^{-5}y^0$

9. $\dfrac{10^{-2}x^{-4}}{y^0}$

10. $\dfrac{3^{-4}x^0}{y^{-8}}$

11. $\dfrac{14x^0y^{-2}}{2^{-1}z^{-3}}$

12. $\dfrac{5^2y^{-10}}{5^{-1}z^0x^{-5}}$

13. $(x^7)^{-5}$

14. $9^{-12} \bullet 9^4$

15. $\dfrac{y^2 \bullet y^4}{y^3}$

16. $(-2x)^4$

17. $\left(\dfrac{x}{11}\right)^{-2}$

18. $\left(\dfrac{1}{3y^3}\right)^{-3}$

Answers

E. $\dfrac{1}{3}$ **A.** $\dfrac{1}{512}$

H. $\dfrac{1}{9^8}$ **N.** $\dfrac{125x^5}{y^{10}}$

Y. $27y^9$ **O.** $\dfrac{21}{x^5}$

B. $-\dfrac{1}{216}$ **S.** $\dfrac{y^8}{81}$

H. $16x^4$ **Y.** $\dfrac{1}{144}$

E. $\dfrac{1}{100x^4}$ **M.** $\dfrac{121}{x^2}$

O. 1 **C.** y^3

V. $\dfrac{1}{x^{35}}$ **E.** $\dfrac{1}{16}$

T. $\dfrac{15}{y^2}$ **H.** $\dfrac{28z^3}{y^2}$

7	14	2	5		11	4	13	9		16	1	12	6	18	15	8	17	3	10

6.2 Start Thinking

The inverse of a mathematical operation reverses, or "un-does," the operation. For example, addition is the inverse of subtraction because addition reverses subtraction.

What is the inverse of x^2? Give examples for two values of x. What is the inverse of x^3?

6.2 Warm Up

Use the Pythagorean Theorem to find the hypotenuse *c* of a right triangle with the given leg lengths *a* and *b*. Round your answer to the nearest tenth.

1. $a = 3, b = 3$

2. $a = 9, b = 2$

3. $a = 5, b = 7$

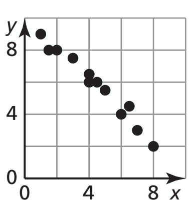
4. $a = 4, b = 5$

5. $a = 2, b = 1$

6. $a = 5, b = 2$

6.2 Cumulative Review Warm Up

Tell whether *x* and *y* show a *positive*, a *negative*, or *no correlation*.

1.


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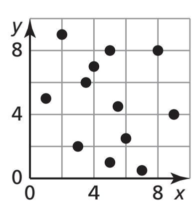

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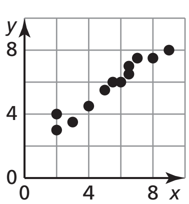

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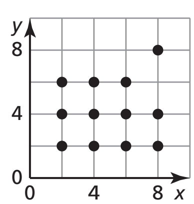


Name____ Date__________

6.2 Practice A

In Exercises 1 and 2, rewrite the expression in rational exponent form.

1. $\sqrt{7}$

2. $\sqrt[4]{13}$

In Exercises 3 and 4, rewrite the expression in radical form.

3. $14^{1/4}$

4. $117^{1/6}$

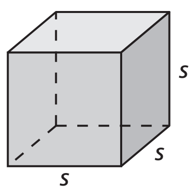
In Exercises 5 and 6, find the indicated real *n*th root(s) of *a*.

5. $n = 3, a = 27$

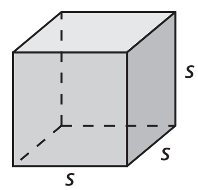
6. $n = 4, a = 16$

In Exercises 7 and 8, find the dimensions of the cube. Check your answer.

7. Volume $= 125 \text{ ft}^3$



8. Volume $= 343 \text{ m}^3$



In Exercises 9–11, evaluate the expression.

9. $\sqrt[3]{-125}$

10. $\sqrt[4]{81}$

11. $\sqrt[4]{-625}$

In Exercises 12 and 13, rewrite the expression in rational exponent form.

12. $\left(\sqrt[4]{14}\right)^3$

13. $\left(\sqrt[3]{-40}\right)^5$

In Exercises 14 and 15, rewrite the expression in radical form.

14. $10^{3/5}$

15. $(-3)^{6/5}$

In Exercises 16–18, evaluate the expression.

16. $81^{3/4}$

17. $25^{3/2}$

18. $(-27)^{2/3}$

19. The area of a square patio is 49^3 square inches. Find the length of one side of the patio.

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6.2 Practice B

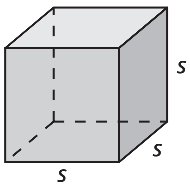
In Exercises 1 and 2, find the indicated *n*th root(s) of *a*.

1. $n = 6, a = 64$

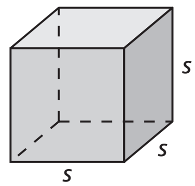
2. $n = 5, a = 243$

In Exercises 3 and 4, find the dimensions of the cube. Check your answer.

3. $\text{Volume} = 729 \text{ cm}^3$



4. $\text{Volume} = 1000 \text{ yd}^3$



In Exercises 5–7, evaluate the expression.

5. $-\sqrt[3]{-512}$

6. $729^{1/6}$

7. $(-625)^{1/4}$

In Exercises 8 and 9, rewrite the expression in rational exponent form.

8. $\left(\sqrt[5]{-53}\right)^4$

9. $\left(\sqrt[4]{110}\right)^7$

In Exercises 10 and 11, rewrite the expression in radical form.

10. $(-34)^{4/9}$

11. $41^{7/4}$

In Exercises 12–17, evaluate the expression.

12. $(-128)^{3/7}$

13. $(-25)^{5/2}$

14. $1000^{4/3}$

15. $\left(\frac{1}{125}\right)^{2/3}$

16. $(343)^{-1/3}$

17. $\left(\frac{1}{64}\right)^{3/2}$

18. The radius of a sphere is given by the equation $r = \left(\frac{3V}{4\pi}\right)^{1/3}$, where V is the volume of the sphere. Find the radius, to the nearest centimeter, of a sphere that has a volume of 268 cubic centimeters. Use 3.14 for π.

19. Use the formula $r = \left(\frac{F}{P}\right)^{1/n} - 1$ to find the annual inflation rate to the nearest tenth of a percent. A rare coin increases in value from $0.25 to $1.50 over a period of 30 years.

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6.2 Enrichment and Extension

Combine Exponent Rules, Radicals, and Rational Exponents

When simplifying with radicals and rational exponents, you must never leave either in the denominator of an expression. To fix this, use the method of rationalizing the denominator. To rationalize, you must mulitply both the numerator and denominator by either the radical in the original denominator or a rational exponent chosen so its product with the original denominator has a whole number exponent.

Example: Simplify (a) $\frac{2}{\sqrt{3}}$ and (b) $\frac{x}{x^{1/3}}$.

a. $\frac{2}{\sqrt{3}} \bullet \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{\sqrt{9}} = \frac{2\sqrt{3}}{3}$

b. $\frac{x}{x^{1/3}} \bullet \frac{x^{2/3}}{x^{2/3}} = \frac{x^{5/3}}{x}$

Simplify the expression. Write your answer using only positive exponents.

1. $\frac{20\sqrt{2}}{2\sqrt{8}}$

2. $\frac{3 - \sqrt{2}}{\sqrt{12}}$

3. $x^{-5/3}$

4. $\left(x^4\right)^{3/8}$

5. $x^3 \bullet x^{-2/3} \bullet x^{1/2}$

6. $\frac{\sqrt[8]{16}}{\sqrt[5]{4}}$

7. $\sqrt[3]{\frac{125}{81}}$

8. $27^{1/2} \bullet 3^{1/2}$

9. $\sqrt[6]{x^{10}}$

10. $\frac{x^2}{x^{1/5}}$

11. $(-27)^{-4/3}$

12. $\frac{2x^{-1/3}}{x^{1/6} \bullet x^{-1/2}}$

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6.2 Puzzle Time

Why Did The Mother Skunk Take Her Baby To See The Doctor?

Write the letter of each answer in the box containing the exercise number.

Rewrite the expression in rational exponent form.

1. $\sqrt{8}$ **2.** $\sqrt[6]{43}$

3. $\left(\sqrt[3]{9}\right)^5$ **4.** $\left(\sqrt[7]{-51}\right)^{11}$

Rewrite the expression in radical form.

5. $19^{1/4}$ **6.** $802^{1/7}$

7. $(-12)^{2/5}$ **8.** $16^{3/2}$

Evaluate the expression.

9. $\sqrt[3]{125}$ **10.** $\sqrt[5]{-32}$ **11.** $243^{1/5}$

12. $(512)^{1/3}$ **13.** $64^{5/6}$ **14.** $(256)^{3/4}$

15. $(-128)^{5/7}$ **16.** $(-125)^{4/3}$ **17.** $\left(\frac{1}{64}\right)^{5/6}$

18. $\left(\frac{16}{625}\right)^{3/4}$ **19.** $(343)^{-2/3}$ **20.** $\frac{1}{81^{1/4}}$

21. The volume of a number cube is 3^6 cubic millimeters. Find the length of one side of the number cube.

Answers

O. 64	**S.** $\left(\sqrt{16}\right)^3$
E. $(-51)^{11/7}$	**C.** $\frac{1}{32}$
W. 8	**U.** $9^{5/3}$
I. $\frac{1}{49}$	**O.** 625
U. $\left(\sqrt[5]{-12}\right)^2$	**B.** -2
A. $8^{1/2}$	**F.** 32
O. $\frac{8}{125}$	**T.** $43^{1/6}$
E. 5	**S.** -32
R. $\frac{1}{3}$	**D.** 9
T. $\sqrt[4]{19}$	**A.** $\sqrt[7]{802}$
O. 3	

10	4	17	6	3	15	9		19	5		12	1	8	
16	7	2		18	13		14	21	11	20				

6.3 Start Thinking

Use the function $f(x) = 3^x$ to complete the table.

x	0	1	2	3	4
$f(x)$					

Find the difference between each $f(x)$-value. What is the pattern? How does this relate to the function? If the function is changed to $f(x) = 2^x$, is the pattern still the same?

6.3 Warm Up

Determine whether the equation represents a linear function. Explain.

1. $y = \sqrt{2x + 3}$

2. $y = 2x^3 - 3x + 2$

3. $y = x + 1$

4. $y = x^2 - 1$

5. $y = 1 - \frac{1}{2}x$

6. $y = \frac{1}{2}x - 3$

6.3 Cumulative Review Warm Up

Solve the system of linear equations.

1. $y = -x + 3$
$-y = x + 3$

2. $2x + 2y = 5$
$-2x - 2y = -5$

3. $3x + 4y = 0$
$-3x - 4y = 0$

4. $6x - 2 = y$
$6x = y$

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6.3 Practice A

In Exercises 1–3, determine whether the equation represents an exponential function. Explain.

1. $y = 9x$

2. $y = 2(3)^x$

3. $y = (-2)^x$

In Exercises 4 and 5, determine whether the table represents a *linear* or an *exponential* function. Explain.

4.

x	y
1	3
2	9
3	27
4	81

5.

x	y
1	4
2	6
3	8
4	10

In Exercises 6 and 7, evaluate the function for the given value of *x*.

6. $y = 2(4)^x;\ x = -2$

7. $f(x) = -3(5)^x;\ x = 3$

In Exercises 8–10, graph the function. Compare the graph to the graph of the parent function. Describe the domain and range of *f*.

8. $f(x) = -2(0.5)^x$

9. $f(x) = -\left(\frac{1}{3}\right)^x$

10. $f(x) = \frac{1}{2}(6)^x$

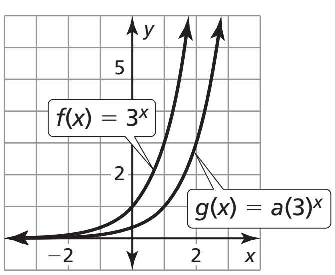
In Exercises 11 and 12, graph the function. Describe the domain and range.

11. $f(x) = 2^x + 3$

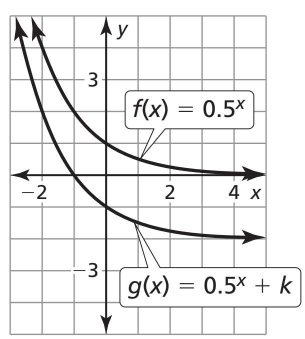
12. $f(x) = 3^{x-2}$

In Exercises 13 and 14, compare the graphs. Find the value of *h*, *k*, or *a*.

13.



14.



15. Graph the function $f(x) = 2^x$. Then graph $g(x) = 3(2)^x$. How are the y-intercept, domain, and range affected by the transformation?

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6.3 Practice B

In Exercises 1–3, determine whether the equation represents an exponential function. Explain.

1. $y = -6^x$ **2.** $y = 5(1)^x$ **3.** $y = 7x^3$

In Exercises 4 and 5, determine whether the table represents a *linear* or an *exponential* function. Explain.

4.

x	y
1	5
2	2
3	−1
4	−4

5.

x	y
1	24
2	12
3	6
4	3

In Exercises 6 and 7, evaluate the function for the given value of *x*.

6. $y = (1.2)^x;\ x = 2$ **7.** $f(x) = \frac{1}{2}(8)^x;\ x = -2$

In Exercises 8–10, graph the function. Compare the graph to the graph of the parent function. Describe the domain and range of *f*.

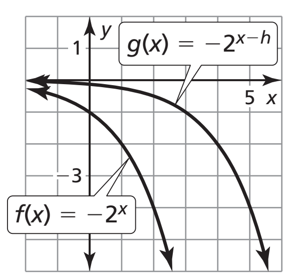
8. $f(x) = 5\left(\frac{1}{4}\right)^x$ **9.** $f(x) = -\frac{1}{3}(3)^x$ **10.** $f(x) = \frac{4}{3}(6)^x$

In Exercises 11 and 12, graph the function. Describe the domain and range.

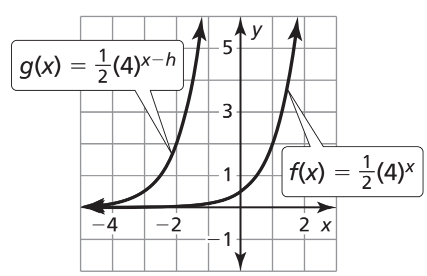
11. $f(x) = -6\left(\frac{1}{3}\right)^{x-1} - 4$ **12.** $f(x) = 2(5)^{x+1} - 3$

In Exercises 13 and 14, compare the graphs. Find the value of *h*, *k*, or *a*.

13.



14.



15. Graph the function $f(x) = 2^x$. Then graph $g(x) = 2^{x-3}$. How are the y-intercept, domain, and range affected by the transformation?

Name ____ Date __________

6.3 Enrichment and Extension

An Investment Opportunity

When you were born, your uncle gave you $5000. Your parents decided to invest the money for you in hopes that someday, you would be able to buy books for your college classes. They went to various banks and were interested in the offers found at two different places. One offer would pay interest on the amount invested at 3.99% compounded annually; the other earned the same interest rate but was compounded quarterly. If the money was invested for 18 years, with which offer do you hope your parents invested your money? How much better is that offer than the lower profit-generating one? Algebraically show how much money you would have in each account at the time you would go to college. To prove this, show your investments graphically over the span of the 18 years.

Name______________________________ Date__________

6.3 Puzzle Time

How Do You Find A Train?

Write the letter of each answer in the box containing the exercise number.

Determine whether the equation represents an exponential function.

1. $y = 2(15)^x$

2. $y = 6(-11)^x$

3.

x	−2	−1	0	1	2
y	1	5	25	125	625

4.

x	7	8	9	10	11
y	2	6	10	14	18

Evaluate the function for the given value of *x*.

5. $y = 4^x;\ x = -2$

6. $y = 3(5)^x;\ x = 2$

7. $y = -7(2)^x;\ x = -5$

8. $f(x) = 0.25^x;\ x = -4$

9. $f(x) = -\frac{1}{6}(6)^x;\ x = 3$

10. $y = \frac{1}{9}(27)^x;\ x = \frac{2}{3}$

Describe the domain and range of the function.

11. $f(x) = 2^x + 3$

12. $f(x) = 5^{x-4}$

13. $y = -\left(\frac{1}{6}\right)^x - 8$

14. $y = 9^{x+1} - 1$

15. The function $y = 3(2)^x$ represents the population of bees in the bee hive, where *x* represents the number of days. How many bees are in the bee hive after 4 days?

Answers

S. no

L. yes

W. −36

T. $-\frac{7}{32}$

I. 48

F. $\frac{1}{16}$

A. 256

T. 1

C. 75

O. all real numbers; $y > -1$

R. all real numbers; $y < -8$

K. all real numbers; $y > 0$

O. all real numbers; $y > 3$

5	11	3	1	14	9		15	7	4		10	13	8	6	12	2

6.4 Start Thinking

When a department store offers a clothing discount of 20%, the customer pays 80% of the original price. Likewise, if a store is selling an item at a 50% discount, the customer pays 50% of the original price. Explain how to determine the percent a customer pays if the discount is 35%.

If a certain state's population has decreased by 4% in the past year, what percent of the original population is still living in that state? If the state's population has increased by 4% instead, what percent of the original population is still living in the state?

6.4 Warm Up

Use the simple interest formula, $I = Prt$, where I = interest, P = principle, r = interest rate, and t = time in years, to find the interest.

1. $P = \$3500$
$r = 5\%$
$t = 3$ years

2. $P = \$250{,}000$
$r = 7\%$
$t = 8$ years

3. $P = \$6000$
$r = 9\%$
$t = 3$ years

4. $P = \$15{,}000$
$r = 7\%$
$t = 7$ years

6.4 Cumulative Review Warm Up

Solve the equation. Check your solutions.

1. $|5n + 12| = |n|$

2. $|3b + 9| = |b + 5|$

Name____ Date__________

6.4 Practice A

In Exercises 1–3, identify the initial amount *a* and the rate of growth *r* (as a percent) of the exponential function. Evaluate the function when *t* = 5. Round your answer to the nearest tenth.

1. $y = 50(1 + 0.25)^t$ **2.** $y = 172(1 + 0.3)^t$ **3.** $y = 1000(1.75)^t$

In Exercises 4 and 5, write a function that represents the situation.

4. Profits of \$100,000 increase by 15% each year.

5. College enrollment of 41,000 increases by 7% every year.

6. The number of food trucks in a city has been increasing by 50% annually. There were two food trucks in the year 2010.

a. Write an exponential growth function that represents the number of food trucks *t* years after 2010.

b. How many food trucks will there be in the year 2030? Does this sound reasonable? Explain.

In Exercises 7–9, identify the initial amount *a* and the rate of decay *r* (as a percent) of the exponential function. Evaluate the function when *t* = 3. Round your answer to the nearest tenth.

7. $y = 12(1 - 0.35)^t$ **8.** $y = 360(1 - 0.9)^t$ **9.** $h(t) = 550(0.4)^t$

In Exercises 10 and 11, write a function that represents the situation.

10. A school population of 1200 decreases by 6% each year.

11. A stock valued at \$49.50 decreases in value by 7% each year.

In Exercises 12 and 13, determine whether the table represents an *exponential growth function*, an *exponential decay function*, or *neither*. Explain.

12.

x	0	1	2	3
y	4	12	36	108

13.

x	0	1	2	3
y	200	100	50	25

In Exercises 14–16, determine whether the function represents *exponential growth* or *exponential decay*. Identify the percent rate of change.

14. $y = 3(0.4)^t$ **15.** $y = 18(1.3)^t$ **16.** $y = 41(1.07)^t$

Name ____ Date _________

6.4 Practice B

In Exercises 1–3, identify the initial amount *a* and the rate of growth *r* (as a percent) of the exponential function. Evaluate the function when $t = 5$. Round your answer to the nearest tenth.

1. $f(t) = 220(1.015)^t$

2. $p(t) = 5.5(1.5)^t$

3. $h(t) = 2.5^t$

In Exercises 4 and 5, write a function that represents the situation.

4. A college's tuition of $135 per credit hour increases by 5% each year.

5. A bee population of 3000 increases by 40% every year.

In Exercises 6–8, identify the initial amount *a* and the rate of decay *r* (as a percent) of the exponential function. Evaluate the function when $t = 3$. Round your answer to the nearest tenth.

6. $f(t) = 1420(0.895)^t$

7. $y = \left(\frac{3}{5}\right)^t$

8. $y = 9.2\left(\frac{1}{3}\right)^t$

In Exercises 9 and 10, write a function that represents the situation.

9. A $25,000 car decreases by 16.7% each year.

10. A company's annual revenue of $487,000 decreases by 4.2% each year.

In Exercises 11 and 12, determine whether the table represents an *exponential growth function*, an *exponential decay function*, or *neither*. Explain.

11.

x	2	4	6	8
y	5	10	15	20

12.

x	1	5	9	13
y	81	54	36	24

13. The table shows the total numbers of shares of an initial public offering purchased *t* days after it opens on the stock market.

x	1	2	3	4
y	6250	2500	1000	400

a. Determine whether the table represents an exponential growth function, an exponential decay function, or neither.

b. How many shares were purchased after the stock had been opened for 6 days?

In Exercises 14–16, rewrite the function to determine whether it represents *exponential growth* or *exponential decay*.

14. $y = (0.3)^{t-2}$

15. $y = 3(1.6)^{4t}$

16. $y = 9(0.68)^{t/3}$

Name____ Date__________

6.4 Enrichment and Extension

Using Your Knowledge of Exponential Growth and Decay

Complete the following exercises. You may have to estimate some of the outcomes.

1. A biologist is researching a newly discovered species of bacteria. At time $t = 0$ hours, he puts 100 bacteria into what he has determined to be a favorable growth medium. Five hours later, he measures 400 bacteria. Assuming exponential growth, what is the growth rate for the bacteria?

2. The number of bacteria in a colony is growing exponentially. At the start you have 10 bacteria, and after 4 hours, there are 300. Write an exponential function to model the population of bacteria after x hours. What is the growth rate of the bacteria? How many bacteria were there after 20 hours? When were there 1000 bacteria?

3. Laura invested $2000 in a three-year CD that pays 4% compounded annually. What is the compound interest rate and amount that will be in the bank after 3 years?

4. A certain radioactive substance has a half-life of about 2650 years. Write an equation describing the situation with the original quantity of the substance being 200 milligrams. How much will remain after 500 years? In how many years will there be half of the original left?

5. The price of computers drops drastically as technology improves. Every year the cost of computing decreases by 20%. If a laptop costs $1200 today, how much will it cost in 1 year? in 2 years?

6. The baseball cards LeRoy started collecting when he was 7 years old appreciated in value at a rate of 5% per year. If his collection was worth $215.75 when he was 10, how much were they worth when he was 15 years old?

7. Assume that you are observing the behavior of cell duplication in a lab. In one experiment, you started with one cell and the cells doubled every minute. Write an expression to determine the population of cells after 1 hour.

8. Hospitals utilize the radioactive substance iodine-131 in the diagnosis of conditions of the thyroid gland. The half-life of iodine-131 is 8 days. If a hospital acquires 6 grams of iodine-131, how much of this sample will remain after 20 days? About how long will it be until only 0.01 gram remains?

Name ______________________________ Date __________

6.4 Puzzle Time

What Looks Like Half A Lemon?

Write the letter of each answer in the box containing the exercise number.

Evaluate the function when $t = 4$. Round your answer to the nearest hundredth.

1. $y = 275(1 + 0.85)^t$

2. $y = 9(1 + 0.03)^t$

3. $f(t) = 16(1.7)^t$

4. $p(t) = 8.21(1.09)^t$

Evaluate the function when $t = 7$. Round your answer to the nearest hundredth.

5. $y = 725(1 - 0.1)^t$

6. $g(t) = 360(0.45)^t$

7. $r(t) = \left(\frac{11}{12}\right)^t$

8. $h(t) = 0.8\left(\frac{3}{5}\right)^t$

Write a function that represents the situation.

9. A \$33,000 vehicle decreases in value by 27% each year.

10. Your hourly wage of \$9.56 increases by 3% each year.

11. The amount of five fruit flies increases by 12.5% each day.

12. A \$4000 deposit that earns 2% annual interest compounded semiannually after t years.

Answers

L. 11.59

T. 0.02

E. 10.13

R. 0.54

H. 346.77

E. 1.35

A. 3221.21

H. 133.63

F. $f(t) = 5(1 + 0.125)^t$

H. $f(t) = 9.56(1.03)^t$

T. $f(t) = 4000(1.01)^{2t}$

O. $f(t) = 33,000(1 - 0.27)^t$

8	5	2		9	12	3	6	7		10	1	4	11

6.5 Start Thinking

The exponential equation $3 \bullet 7^x = -2 \bullet 8^x$ can be graphed on a graphing calculator using a system of equations:

$y = 3 \bullet 7^x$

$y = -2 \bullet 8^x$

Explain how to get the system shown above from the equation. Graph the system. Is there an intersection? Use the equations to explain why.

6.5 Warm Up

Simplify.

1. $\dfrac{t^6u^3}{t^3u}$

2. $\dfrac{g^7h^3m}{hg^6}$

3. $\left(\dfrac{3a^2b^6}{2}\right)^3$

4. $\left(\dfrac{f^{-4}g^3}{h^{-5}}\right)^1$

5. $\dfrac{m^3p^3}{mp}$

6. $\dfrac{c^5d^3f^4}{cd^5f^2}$

6.5 Cumulative Review Warm Up

Solve the inequality. Graph the solution.

1. $5x > 5$

2. $-21 \geq 7n$

3. $\dfrac{x}{5} < -3$

4. $25 \leq \dfrac{5}{4}w$

5. $-5t > 10$

6. $-8 \leq -4z$

Name ____ Date _________

6.5 Practice A

In Exercises 1–9, solve the equation. Check your solution.

1. $3^{4x} = 3^{12}$ **2.** $2^{x+3} = 2^5$ **3.** $5^{3x} = 5^{2x-7}$

4. $3^x = 27$ **5.** $5^x = 625$ **6.** $11^{x-4} = 121^x$

7. $\left(\frac{1}{3}\right)^x = 81$ **8.** $\frac{1}{125} = 5^{2x+7}$ **9.** $7^{5-4x} = \frac{1}{343}$

10. Describe and correct the error in solving the exponential equation.

✗
$$\left(\frac{1}{6}\right)^{3x-1} = 36^{x-7}$$
$$\left(6^{-1}\right)^{3x-1} = \left(6^{-2}\right)^{x-7}$$
$$-3x + 1 = -2x + 14$$
$$x = -13$$

In Exercises 11–16, use a graphing calculator to solve the equation.

11. $5^{x-1} = 10$ **12.** $3^{x+5} = 7$ **13.** $\left(\frac{1}{3}\right)^{6x+1} = -5$

14. $\left(\frac{1}{4}\right)^{x+2} = 9$ **15.** $3^{x-5} = 3x - 4$ **16.** $4x + 1 = 5^{x-3}$

In Exercises 17–19, solve the equation using the Property of Equality for Exponential Equations.

17. $40 \bullet 5^{x-2} = 200$ **18.** $8 \bullet 2^{x+6} = 32$ **19.** $3\left(4^{-3x-1}\right) = 48$

20. A bacterial culture triples in size every hour. You begin observing the number of bacteria 2 hours after the culture is prepared. The amount y of bacteria x hours after the culture is prepared is represented by $y = 162\left(3^{x-2}\right)$. When will there be 8100 bacteria?

In Exercises 21–23, solve the equation.

21. $2^{3x-6} = 8^{x-2}$ **22.** $9^{3x-2} = 27^{2x-2}$ **23.** $2^{4(x-3)} = 16^{x+1}$

In Exercises 24 and 25, solve the equation.

24. $7^{x+3} = \sqrt{7}$ **25.** $\left(\sqrt[4]{10}\right)^x = 10^{3x-1}$

Name____ Date__________

6.5 Practice B

In Exercises 1–9, solve the equation. Check your solution.

1. $3^{8x} = 3^{5x-6}$

2. $4^{x} = 2^{5x+3}$

3. $8^{5x} = 4^{4x+7}$

4. $25^{x-2} = 125^{3x+1}$

5. $9^{x-6} = 729^{3(x+2)}$

6. $4^{6(-x+2)} = 8^{-3x-4}$

7. $\left(\frac{1}{8}\right)^{2x+4} = 16^{4-x}$

8. $\left(\frac{2}{3}\right)^{x+8} = \left(\frac{3}{2}\right)^{2x-5}$

9. $\left(\frac{5}{4}\right)^{3x+5} = \left(\frac{16}{25}\right)^{-4x}$

10. Describe and correct the error in solving the exponential equation.

$$\begin{aligned} \left(\frac{1}{16}\right)^{3x} &= 64^{x-4} \\ \left(4^{-2}\right)^{3x} &= \left(8^{2}\right)^{x-4} \\ -6x &= 2x - 8 \\ x &= 1 \end{aligned}$$

In Exercises 11–16, use a graphing calculator to solve the equation.

11. $4^{-x+2} = -\frac{1}{3}x + 5$

12. $\frac{1}{2}x + 3 = \left(\frac{1}{5}\right)^{2x+1}$

13. $6^{x} = 4^{-x+3}$

14. $5^{x-4} = 3^{-x}$

15. $3^{x+2} = -4^{-x+1}$

16. $3^{-x-5} = 2^{x+3}$

17. A bread dough doubles in size every hour. You begin measuring the volume of the dough 1 hour after the dough is prepared. The volume y (in cubic inches) of the dough x hours after the dough is prepared is represented by $y = 35\left(2^{x-1}\right)$.

When will the volume of the dough be 4200 cubic inches?

In Exercises 18–20, solve the equation.

18. $125^{x-1} = 5^{3x-2}$

19. $8^{2x+1} = 2^{3(2x+1)}$

20. $3^{8(2x-1)} = 81^{4x-2}$

21. You deposit $750 in a savings account that earns 4% annual interest compounded yearly. Write and solve an exponential equation to determine when the balance of the account will be $1000.

In Exercises 22 and 23, solve the equation.

22. $\left(\sqrt[5]{3}\right)^{x} = 3^{3x-5}$

23. $\left(\sqrt[6]{2}\right)^{2x} = \left(\sqrt[4]{2}\right)^{x-3}$

Name ____ Date __________

6.5 Enrichment and Extension

Challenge: Solve Exponential Equations

Solve the exponential equation.

1. $9^{x-3} = \dfrac{\left(\frac{1}{27}\right)^{x}}{3^{x+6}}$

2. $4^{2x+3} \bullet \left(\frac{1}{4}\right)^{3x} = 4^{x-7}$

3. $\dfrac{\left(\frac{1}{6}\right)^{x}}{216^{3x-4}} = 1$

4. $\dfrac{\left(\frac{1}{27}\right)^{-x}}{\left(\frac{1}{3}\right)^{x-5}} = 9^{2x}$

5. $\dfrac{32^{x-3}}{16^{3x+1}} = 4^{x} \bullet 2^{-3x+7}$

6. $\dfrac{5^{-3x+2}}{125} = 25^{x} \bullet 5^{3x}$

7. $\dfrac{\left(\frac{1}{2}\right)^{x}}{16^{4x+2}} = 0$

Name____ Date__________

6.5 Puzzle Time

How Did The Beetle Uncover The Ant's Secret Plan?

Circle the letter of each correct answer in the boxes below. The circled letters will spell out the answer to the riddle.

Solve the equation. Check your solution.

1. $7^{8x} = 7^{16}$

2. $9^{x-5} = 9^{11}$

3. $4^{10x} = 4^{6x+12}$

4. $8^{5x} = 8^{x-8}$

5. $3^{x} = 81$

6. $6^{x-3} = 36^{x}$

7. $343^{x} = 7^{x-8}$

8. $125^{x} = 5^{x+12}$

9. $\left(\frac{1}{2}\right)^{x} = 128$

10. $\frac{1}{256} = 4^{5x+1}$

11. $100^{x-7} = \left(\frac{1}{1000}\right)^{x-7}$

12. $\left(\frac{1}{243}\right)^{x-3} = 27^{-2x+3}$

Use a graphing calculator to solve the equation. Round your answer to the nearest hundredth.

13. $6^{x+5} = 9$

14. $\left(\frac{1}{4}\right)^{6x-2} = 7^{x}$

15. $4x - 3 = 8^{x-2}$

16. $2^{x-4} = 3^{-x}$

D	I	F	T	G	K	B	V	U	G	H	G	L	B	E	C
15	−4	−28	16	−24	8	−3.77	24	−2	7	−15	4	−19	0.48	−7	−0.01
D	**K**	**I**	**N**	**T**	**O**	**S**	**P**	**P**	**Q**	**H**	**B**	**O**	**W**	**N**	**E**
3	28	−6	−0.55	6	19	−3	−8	0.77	24	−1	−0.48	0.27	0.01	2	1.55

6.6 Start Thinking

Find the result of $256 \bullet \frac{1}{4}$. Continue multiplying each result by $\frac{1}{4}$ until you reach a result of 1. Write each result in a list.

Is the list getting larger or smaller? Why? If the list continues, what will the numbers look like? Is it possible to get negative numbers in the list? Why or why not?

6.6 Warm Up

Find the common difference in the sequence.

1. $-2, 0, 2, 4, \ldots$

2. $-9, -8, -7, -6, \ldots$

3. $0.03, 0.09, 0.15, 0.21, \ldots$

4. $12, 10, 8, 6, \ldots$

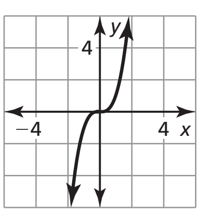
5. $2, 3\frac{1}{4}, 4\frac{1}{2}, 5\frac{3}{4}, \ldots$

6. $-4, -7, -10, -13, \ldots$

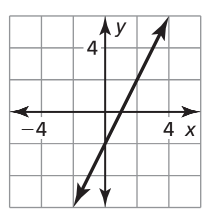
6.6 Cumulative Review Warm Up

Determine whether the graph represents a *linear* or *nonlinear* function. Explain.

1.



2.



Name______________________________ Date__________

6.6 Practice A

In Exercises 1–3, find the common ratio of the geometric sequence.

1. 2, 6, 18, 54, …

2. 135, 45, 15, 5, …

3. 7, −14, 28, −56, …

In Exercises 4–6, determine whether the sequence is *arithmetic*, *geometric*, or *neither*. Explain your reasoning.

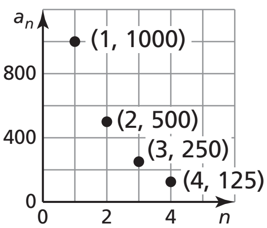
4. 1, 4, 9, 16, …

5. 12, 17, 22, 27, …

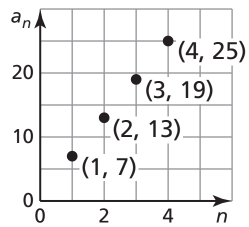
6. 4, −12, 36, −108, …

In Exercises 7 and 8, determine whether the graph represents an *arithmetic sequence*, a *geometric sequence*, or *neither*. Explain your reasoning.

7.



8.



In Exercises 9 and 10, write the next three terms of the geometric sequence. Then graph the sequence.

9. 3, 15, 75, 375, …

10. 1024, −256, 64, −16, …

In Exercises 11–14, write an equation for the *n*th term of the geometric sequence. Then find a_6.

11. 3, 6, 12, 24, …

12. 0.375, 3, 24, 192, …

13.

n	1	2	3	4
a_n	0.0124	1.24	124	12,400

14.

n	1	2	3	4
a_n	−1024	128	−16	2

15. A digital city map displays an area of 544 square units. After you zoom in once, the area is 272 square units. After you zoom in a second time, the area is 136 square units. What is the area after you zoom in five times?

16. What is the 8th term of the geometric sequence where $a_2 = 20$ and $r = 5$?

Name ______________________________ Date __________

6.6 Practice B

In Exercises 1–3, find the common ratio of the geometric sequence.

1. 5, 20, 80, 320, … **2.** 144, −72, 36, −18, … **3.** 24, 84, 294, 1029, …

In Exercises 4–7, determine whether the sequence is *arithmetic*, *geometric*, or *neither*. Explain your reasoning.

4. 2.786, 27.86, 278.6, 2786, … **5.** 86, 71, 56, 41, …

6. 4, −10, 16, −28, … **7.** 112, −28, 7, $-\frac{7}{4}$, …

In Exercises 8 and 9, write the next three terms of the geometric sequence. Then graph the sequence.

8. −2, −12, −72, −432, … **9.** $\frac{54}{25}$, $\frac{18}{5}$, 6, 10, …

In Exercises 10–13, write an equation for the *n*th term of the geometric sequence. Then find a_6.

10. $\frac{3}{125}$, $\frac{3}{25}$, $\frac{3}{5}$, 3, … **11.** 0.2, 1.6, 12.8, 102.4, …

12.

n	1	2	3	4
a_n	2436	−243.6	24.36	−2.436

13.

n	1	2	3	4
a_n	−1458	−162	−18	−2

14. An archery competition begins with 256 competitors. After the first round, one-fourth of the competing group remains. After the second round, one-fourth of the now smaller competing group remains. The last round is when there are fewer than five members in the competing group.

a. Which round is the last round?

b. How many competitors are in the last round?

15. What is the 10th term of the geometric sequence where $a_3 = \frac{8}{3}$ and $r = \frac{2}{3}$?

16. Find the sum of the terms of the geometric sequence

$$1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots.$$

Explain your reasoning.

Name____ Date__________

6.6 Enrichment and Extension

Geometric Series

A geometric series is the sum of a certain number of terms of a geometric sequence. Let S be sum of any geometric series. You can either find the sum of a partial geometric series or an infinate geometric series.

For a partial sum of a geometric sequence with first term a_1 and common ratio r, the sum of the first n terms is given by the formula $S = \dfrac{a_1(1 - r^n)}{1 - r}$. The sum S of an infinite geometric series with $|r| < 1$ is given by $S = \dfrac{a_1}{1 - r}$. If $|r| \geq 1$, series will have no sum.

Example: Find the sum of the first four terms of the geometric series $-3 + 1 - \dfrac{1}{3} + \dfrac{1}{9} + \cdots$.

$$a_1 = -3,\ n = 4,\ r = -\frac{1}{3};\ \text{So, } S = \frac{-3\left(1 - \left(-\frac{1}{3}\right)^4\right)}{1 - \left(-\frac{1}{3}\right)} = -\frac{20}{9}.$$

In Exercises 1–3, find the partial sum of the geometric series.

1. $a_1 = 3, n = 5, r = -2$

2. $a_1 = 8, n = 4, r = \frac{2}{5}$

3. $5 + \frac{5}{2} + \frac{5}{4} + \frac{5}{8} + \cdots, n = 10$

In Exercises 4–6, find the infinite sum of the geometric series, if it exists.

4. $3 - \frac{3}{2} + \frac{3}{4} - \frac{3}{8} + \cdots$

5. $-1 - 4 - 16 - 32 - \cdots$

6. $x + \dfrac{x}{2} + \dfrac{x}{4} + \cdots$

7. Find the first term of the series if $S_5 = 27$ and $r = -3$.

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6.6 Puzzle Time

How Does A Penguin Make Pancakes?

Write the letter of each answer in the box containing the exercise number.

Find the common ratio of the geometric sequence.

1. 5, 20, 80, 320, …

2. 49, 7, 1, $\frac{1}{7}$, …

3. $\frac{2}{9}$, −2, 18, −162, …

4. 0.023, 0.23, 2.3, 23, …

5. $\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, -\frac{1}{16}$, …

6. 6, 18, 54, 162, …

Write the next three terms of the geometric sequence.

7. 192, 48, 12, 3, …

8. 6, 12, 24, 48, …

9. −100, 10, −1, 0.1, …

10. −500, −100, −20, −4, …

Write an equation for the *n*th term of the geometric sequence. Then find a_6.

11. 3, 12, 48, 192, …

12. 2.88, 1.44, 0.72, 0.36, …

13. $-\frac{1}{81}, -\frac{1}{27}, -\frac{1}{9}, -\frac{1}{3}$, …

14. −256, 64, −16, 4, …

15. There were 32 chess players in the competition. After the first round, 16 players remained. After the second round, 8 players remained. Write an equation for the *n*th term of the geometric sequence. How many players remained after the fifth round?

Answers

S. 3

I. $\frac{1}{7}$

H. 2

L. 10

R. 3072

E. 4

I. −3

F. 0.09

T. $-\frac{1}{2}$

P. $\frac{1}{4}$

T. −9

P. −0.01, 0.001, −0.0001

S. $\frac{3}{4}, \frac{3}{16}, \frac{3}{64}$

W. $-\frac{4}{5}, -\frac{4}{25}, -\frac{4}{125}$

I. 96,192,384

10	8	3	15		2	5	7		12	4	13	9	14	1	11	6

6.7 Start Thinking

Use the sequence $-12, -14, -16, -18, \ldots$ to complete the table.

Term	Term number	Common difference
-12		
-14		
-16		
-18		

The recursive rule $a_1 = -2, a_n = a_{n-1} - 2$ represents the sequence above. Explain this rule as it relates to the sequence. Can you use this rule to determine the term preceeding -12? If so, what is it?

6.7 Warm Up

Find the next three terms in the sequence.

1. $1, -16, -33, -50, \ldots$

2. $6, 7, 8, 9, \ldots$

3. $-39, -13, -\frac{13}{3}, -\frac{13}{9}, \ldots$

4. $0.5, 1.5, 2.5, 3.5, \ldots$

5. $-1, -8, -64, -512, \ldots$

6. $5, 16, 27, 38, \ldots$

6.7 Cumulative Review Warm Up

Write a linear function *f* with the given values.

1. $f(3) = -3, f(2) = 0$

2. $f(-3) = 1, f(7) = -4$

3. $f(-2) = 0, f(14) = 4$

4. $f(3) = -2, f(5) = 4$

5. $f(-2) = 15, f(0) = 9$

6. $f(1) = 0.3, f(0) = 2.3$

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6.7 Practice A

In Exercises 1 and 2, determine whether the recursive rule represents an *arithmetic sequence* or *geometric sequence*.

1. $a_1 = 3;\ a_n = a_{n-1} + 4$

2. $a_1 = 3;\ a_n = 9a_{n-1}$

In Exercises 3–6, write the first six terms of the sequence. Then graph the sequence.

3. $a_1 = 0;\ a_n = a_{n-1} + 3$

4. $a_1 = 18;\ a_n = a_{n-1} - 8$

5. $a_1 = 1;\ a_n = 5a_{n-1}$

6. $a_1 = 4;\ a_n = 2.5a_{n-1}$

In Exercises 7 and 8, write a recursive rule for the sequence.

7.

n	1	2	3	4
a_n	4	28	196	1372

8.

n	1	2	3	4
a_n	6	11	16	21

In Exercises 9 and 10, write an explicit rule for the recursive rule.

9. $a_1 = -10;\ a_n = a_{n-1} + 5$

10. $a_1 = 14;\ a_n = -2a_{n-1}$

In Exercises 11 and 12, write a recursive rule for the explicit rule.

11. $a_n = 5(2)^{n-1}$

12. $a_n = -7n + 3$

In Exercises 13 and 14, graph the first four terms of the sequence with the given description. Write a recursive rule and an explicit rule for the sequence.

13. The first term of the sequence is 8. Each term of the sequence is 12 more than the preceding term.

14. The first term of the sequence is 81. Each term of the sequence is one-third the preceding term.

In Exercises 15 and 16, write a recursive rule for the sequence. Then write the next two terms of the sequence.

15. 3, 5, 8, 13, 21, …

16. 24, 20, 4, 16, –12, …

17. Write the first five terms of the sequence $a_1 = 4;\ a_n = \frac{1}{2}a_{n-1} + 6$. Determine whether the sequence is *arithmetic*, *geometric*, or *neither*. Explain your reasoning.

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6.7 Practice B

In Exercises 1 and 2, determine whether the recursive rule represents an *arithmetic sequence* or *geometric sequence*.

1. $a_1 = 5;\ a_n = 12a_{n-1}$

2. $a_1 = 6;\ a_n = a_{n-1} - 3$

In Exercises 3–6, write the first six terms of the sequence. Then graph the sequence.

3. $a_1 = 10;\ a_n = a_{n-1} - 7$

4. $a_1 = 36;\ a_n = -1.5a_{n-1}$

5. $a_1 = 120;\ a_n = \frac{1}{5}a_{n-1}$

6. $a_1 = -6;\ a_n = -3a_{n-1}$

In Exercises 7 and 8, write a recursive rule for the sequence.

7.

n	1	2	3	4
a_n	23	13	3	−7

8.

n	1	2	3	4
a_n	256	128	64	32

In Exercises 9 and 10, write an explicit rule for the recursive rule.

9. $a_1 = 8;\ a_n = -9a_{n-1}$

10. $a_1 = 5;\ a_n = a_{n-1} + 18$

In Exercises 11 and 12, write a recursive rule for the explicit rule.

11. $a_n = 1.2n + 2$

12. $a_n = -76\left(\frac{3}{2}\right)^{n-1}$

In Exercises 13 and 14, graph the first four terms of the sequence with the given description. Write a recursive rule and an explicit rule for the sequence.

13. The first term of the sequence is -2. Each term of the sequence is -5 times the preceding term.

14. The first term of the sequence is 23. Each term of the sequence is 9 less than the preceding term.

In Exercises 15 and 16, write a recursive rule for the sequence. Then write the next two terms of the sequence.

15. $4, -4, 0, -4, -4, \ldots$

16. $100, 20, 5, 4, \frac{5}{4}, \ldots$

17. Write the first five terms of the sequence $a_1 = 3;\ a_n = -a_{n-1} + 5$. Determine whether the sequence is *arithmetic*, *geometric*, *recursive*, or *none of these*. Explain your reasoning.

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6.7 Enrichment and Extension

Summation/Sigma Notation

Summation notation, or *sigma notation*, is a convenient shorthand used to write a concise expression to represent a sum with many terms. To find the sum of an infinite geometric series, first determine a_1, n, and r, and then use the infinite series formula.

Example: Expand the series as a sum of terms. Then evaluate the series.

1. $\sum_{n=1}^{5} 2^n = 2^1 + 2^2 + 2^3 + 2^4 + 2^5 = 62$

2. $\sum_{n=1}^{\infty} 3\left(\frac{1}{2}\right)^{n-1} = 3 + \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \cdots = \frac{3}{1 - \frac{1}{2}} = 6$

Expand the series as a sum of terms, if necessary. Then evaluate the series.

1. $\sum_{m=1}^{6} (200 - m)$

2. $\sum_{k=1}^{4} k(k - 1)$

3. $\sum_{n=1}^{6} 3^n$

4. $\sum_{n=1}^{4} 3n^2 - 2$

5. $\sum_{p=1}^{5} 2p - 3$

6. $\sum_{n=1}^{9} \left(\frac{1}{2}\right)^{n-1}$

7. $\sum_{n=1}^{7} 2^{n-1}$

8. $\sum_{n=1}^{\infty} 2\left(\frac{1}{3}\right)^{n-1}$

9. $\sum_{n=1}^{\infty} 5 \bullet 0.2^{n-1}$

Name____ Date__________

6.7 Puzzle Time

What Do Cats Read For Current Events?

Write the letter of each answer in the box containing the exercise number.

Write the first six terms of the sequence.

1. $a_1 = 1, a_n = a_{n-1} + 3$

2. $a_1 = 9, a_n = a_{n-1} - 6$

3. $a_1 = 4, a_n = 2a_{n-1}$

4. $a_1 = -6, a_n = -\frac{1}{2}a_{n-1}$

Write a recursive rule for the sequence.

5. 7, 15, 23, 31, 39, …

6. 625, 125, 25, 5, 1, …

7. 0, −8, −16, −24, −32, …

8. 9, −18, 36, −72, 144, …

Write an explicit rule for the recursive rule.

9. $a_1 = -2, a_n = a_{n-1} + 2$

10. $a_1 = 14, a_n = 0.5a_{n-1}$

11. $a_1 = -3, a_n = 6a_{n-1}$

12. $a_1 = 5, a_n = a_{n-1} + 16$

Write a recursive rule for the explicit rule.

13. $a_n = 8(4)^{n-1}$

14. $a_n = -5n + 7$

15. $a_n = (-10)^{n-1}$

16. $a_n = 12n - 18$

17. The first term of a sequence is 6. Each term of the sequence is 12 less than the preceding term. Write a recursive rule for the sequence.

Answers

H. 4, 8, 16, 32, 64, 128

E. 1, 4, 7, 10, 13, 16

A. 9, 3, −3, −9, −15, −21

I. $-6, 3, -\frac{3}{2}, \frac{3}{4}, -\frac{3}{8}, \frac{3}{16}$

E. $a_1 = 0, a_n = a_{n-1} - 8$

M. $a_1 = 2, a_n = a_{n-1} - 5$

E. $a_1 = 625, a_n = \frac{1}{5}a_{n-1}$

D. $a_1 = 6, a_n = a_{n-1} - 12$

S. $a_1 = 9, a_n = -2a_{n-1}$

L. $a_1 = -6, a_n = a_{n-1} + 12$

P. $a_1 = 8, a_n = 4a_{n-1}$

R. $a_1 = 1, a_n = -10a_{n-1}$

T. $a_1 = 7, a_n = a_{n-1} + 8$

Y. $a_n = 2n - 4$

A. $a_n = -3(6)^{n-1}$

W. $a_n = 16n - 11$

P. $a_n = 14(0.5)^{n-1}$

5	3	7		17	11	4	16	9		14	1	12	8	10	2	13	6	15

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Chapter 6 Cumulative Review

Solve the equation. Check your answer.

1. $5w - 10 = 2w + 2$

2. $6\pi + 5x = \pi$

3. $3(2x - 1) = -2x + 61$

Solve the inequality. Graph the solution, if possible.

4. $|3x - 9| < 36$

5. $\frac{|5x + 10|}{-3} + 8 \le -2$

6. $3|7 - 4x| < -11$

7. The function $f(x) = 125 + 50x$ represents the amount of money a mechanic charges in dollars for x hours of labor and \$125 for parts.

a. What is the total bill for 3 hours of labor?

b. How many hours did it take if the bill is \$325?

Graph the linear equation.

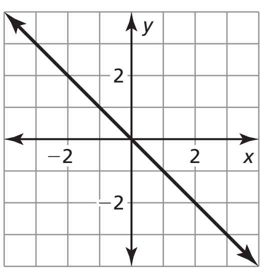
8. $y = \frac{2}{3}x - 1$

9. $x = -3$

10. $12x - 6y = 18$

Write an equation of the line in slope-intercept form.

11.



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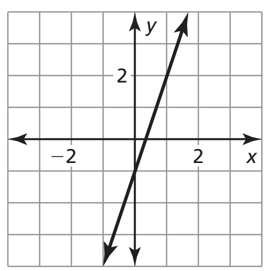

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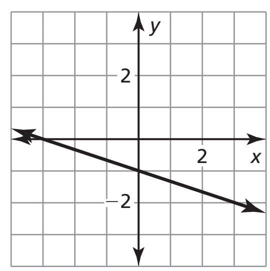

Write an equation in point-slope form of the line that passes through the given point and has the given slope.

14. $(4, 1), (7, 2)$

15. $(-6, -3), (-1, -2)$

16. $(0, 2), (5, 2)$

Write an equation of the line in point-slope form that passes through the given point and is perpendicular to the given line.

17. $(4, 2)$; $y = -3x - 10$

18. $(-1, 0)$; $y = \frac{3}{4}x + 3$

19. $(-4, -4)$; $10x + 5y = 15$

Tell whether *x* and *y* show a *positive*, a *negative*, or *no correlation*.

20.

x	−3	−3	−2	0	2	4	5
y	4	2	1	3	5	4	3

21.

x	−2	−1	−1	0	1	2	2
y	−3	−2	1	0	1	2	3

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Chapter 6 Cumulative Review (continued)

Determine whether the sequence is arithmetic. If so, find the common difference.

22. $1, -4, 7, -10, \ldots$ **23.** $-2, -7, -12, -17, \ldots$ **24.** $2, 4, 8, 16, \ldots$

Graph the function. Describe the domain and range.

25. $y = \begin{cases} 2x + 1, & \text{if } x < -1 \\ 0, & \text{if } x \geq -1 \end{cases}$

26. $y = \begin{cases} x, & \text{if } x < 3 \\ \frac{2}{3}x - 4, & \text{if } x > 3 \end{cases}$

Solve the system of linear equations by graphing, substitution, or elimination.

27. $y = -\frac{1}{2}x - 2$
$y = -\frac{3}{2}x + 2$

28. $8x + 14y = 4$
$-6x - 7y = -10$

29. $y = 5x - 7$
$-3x - 2y = -12$

30. The sum of the digits of a two-digit number is 7. Reversing its digits increases the number by 9. What is the number?

Solve the equation by graphing. Check your solution(s).

31. $9x - 4 = 2 - 3x$ **32.** $|4 - x| = |-6 + x|$

Graph the inequality.

33. $y < \frac{1}{5}x + 2$ **34.** $y \geq -x + 3$ **35.** $2x - 2y \leq -2$

36. You have \$500 in a savings account at the beginning of the summer. You want to have at least \$200 by the end of the summer. You withdraw \$25 each week.

a. Write an inequality that represents this situation.

b. For how many weeks can you withdraw money?

Graph the system of linear inequalities.

37. $x \leq -3$
$y < \frac{5}{3}x + 2$

38. $y \leq \frac{1}{2}x + 2$
$y < -2x - 3$

39. $4x + y < 2$
$y > -2$

Name ____ Date __________

Chapter 6 Cumulative Review (continued)

Evaluate the expression.

40. 2^0 **41.** $(-3)^0$ **42.** 3^{-4} **43.** $\dfrac{(-3)^2}{-8^0}$

Simplify the expression. Write your answer using only positive exponents.

44. w^{-3} **45.** h^0 **46.** $12x^{-5}y^0$

47. $\dfrac{2^{-4}x^2}{z^0}$ **48.** $\dfrac{r^{-7}}{10^{-2}z^{-5}}$ **49.** $\dfrac{17x^{-1}y^{-10}}{7^{-2}z^0}$

Rewrite the expression in rational exponent form.

50. $\sqrt{8}$ **51.** $\sqrt[7]{18}$ **52.** $\sqrt[3]{3}$

Rewrite the expression in radical form.

53. $24^{1/4}$ **54.** $37^{1/10}$ **55.** $140^{1/2}$

Evaluate the expression.

56. $\sqrt[3]{729}$ **57.** $\sqrt[4]{625}$ **58.** $\sqrt[5]{-32}$

59. $512^{2/3}$ **60.** $(-256)^{5/8}$ **61.** $1024^{6/5}$

Use the formula $r = \left(\dfrac{F}{P}\right)^{1/n} - 1$ to find the annual inflation rate to the nearest tenth of a percent.

62. A house increases in value from \$30,000 to \$120,000 over a period of 40 years.

63. The cost of a quart of strawberries increases from \$0.99 to \$3.49 over a period of 25 years.

Determine whether the table represents a *linear* or an *exponential* function.

64.

x	1	2	3	4
y	1	8	27	64

65.

x	−4	0	4	8
y	9	2	−5	−12

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Chapter 6 Cumulative Review (continued)

Evaluate the function for the given value of *x*.

66. $y = 4^x; x = -1$

67. $y = -3(7)^x; x = 4$

68. $f(x) = \frac{1}{4}(2)^x; x = -3$

Identify the initial amount *a* and the rate of growth *r* (as a percent) of the exponential function. Evaluate the function when $t = 4$. Round your answer to the nearest tenth.

69. $y = 250(1 + 0.05)^t$

70. $y = 5(1 + 0.2)^t$

71. $f(t) = 1000(1.002)^t$

72. $p(t) = 3^t$

Write a function that represents the situation.

73. A $20,000 car decreases in value by 15% every year.

74. A newborn baby weighs 8 pounds and increases its weight by 5% every week.

75. A company profit of $1,000,000 decreases by 50% every day.

Solve the equation. Check your solution.

76. $3^{6x} = 3^{18}$

77. $5^{2x+11} = 5^{-7}$

78. $(25)^{3x+6} = (125)^{4x}$

Determine whether the sequence is *arithmetic, geometric,* or *neither*.

79. 180, 90, 45, …

80. 1, 4, 16, 64, …

81. 17, 23, 29, 35, …

Write the next three terms of the geometric sequence.

82. 486, 162, 54, …

83. 6, 12, 24, 48, …

84. 36, 18, 9, $\frac{9}{2}$, …

Write the first six terms of the sequence.

85. $a_1 = 1, a_n = a_{n-1} + 3$

86. $a_1 = 3, a_n = 2a_{n-1}$

87. Write a recursive rule for the number of bacteria at time *t*, if after 1 minute, there is 1 bacterium. After 2 minutes, there are 3 bacteria. After 3 minutes, there are 9 bacteria. After 4 minutes, there are 27 bacteria.