AP Stats - Chapter 6 Notes and Solutions

Let's find the mean of the coin toss situation.
x = heads

$\mu_x = E(x) = \Sigma x_i p_i$

$= (0)(1/8) + (1)(3/8) + (2)(3/8) + (3)(1/8)$

$= 0 + 3/8 + 6/8 + 3/8$

$\mu_x = 1.5$

The mean number of times you will flips heads when tossing a coin three times is 1.5 times.

Standard Deviation of a Discrete Random Variable

variance of a random variable

$Var(x) = \sigma_x^2 = (x_1 - \mu_x)^2 p_1 + (x_2 - \mu_x)^2 p_2 + \ldots.$

$= \Sigma(x_i - \mu_x)\, p_i$

$= (0-1.5)^2(.125) + (1-1.5)^2(.375) + (2-1.5)^2(.375) + (3-1.5)^2(.125)$

$= .28125 + .09375 + .09375 + .28125$

$\sigma_x^2 = .75 \longleftarrow$ variance!

And... Standard deviation of x is

$\sigma_x = \sqrt{0.75}$

$\sigma_x = .8660$

Welcome! Wednesday, January 6th, 2021

Please try this problem...

In an experiment on the behavior of young children, each subject is placed in an area with 5 toys. Past experiments have shown that the probability distribution of the number of X toys PLAYED with by a randomly selected subject is as follows:

# of toys x_i:	0	1	2	3	4	5
Probability p_i:	.03	.16	.30	.23	.17	.11

a. Write the event, "Plays with AT MOST two toys" in terms of X. What is the probability of this event?

$P(X \leq 2) = 0.49$

Homework - AP Stats - January 5th, 2021

Chapter 6.1

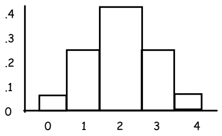
Answers: Page 359, #1,3,5,7

1. a. probability distribution Y = # oF HEAD

value:	0	1	2	3	4
probability:	1/16	4/16	6/16	4/16	1/16

b. histogram

tttt	1
httt thtt ttht ttth	4
hhtt htht htth thht thth tthh	6
hhht hhth hthh thhh	4
hhhh	1



c. $P(x \leq 3) = 15/16 = 0.9375$

3. a. $P(x > 1) = 0.9$

b. The event $x \leq 2$ is "at most two nonword errors."

$P(x \leq 2) = 0.6$ $P(x < 2) = 0.3$

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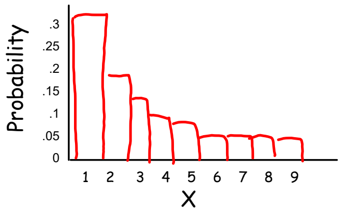
5. a. All of the probabilities are between 0 and 1 and they sum to 1.

b. The histogram below is unimodal and skewed to the right.



c. The event $x \geq 6$ is the event that "the first digit in a randomly chosen record is a 6 or higher."

$P(x \geq 6) = .222$

$P(x \leq 5) = 0.778$

Benford's Law

1	2	3	4	5	6	7	8	9
.301	.176	.125	.097	.079	.067	.058	.051	.046

7. a. the outcomes that make up the event A are 7, 8, and 9.

$P(A) = 0.155$

b. The outcomes that make up the event B are 1,3,5,7, and 9.

$P(B) = \underline{0.609}$

c. the outcomes that make up the event "A or B" are 1,3,5,7,⑧ and 9.

$P(A \text{ or } B) = 0.660$

* This is NOT the same as P(A) + P(B) because the outcomes 7 and 9 are included in both events.

One Problem - January 6th, 2021

Keno is a favorite game in the casinos. Balls numbered 1 to 80 are tumbled in a machine as the bets are placed, then 20 of the balls are chosen at random. Players select numbers by marking a card. The simplest of the many wagers is "Mark 1 number." Your payoff is \$3 on a \$1 bet if the number you select is chosen. Because 20 of 80 numbers are chosen, your probability of winning is 20/80, or 0.25. Let X = the net amount you gain on the single play of a game.

1. Make a table that shows the probability distribution of x.

2. Compute the expected value of X. Explain what this means for the player.

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1. Make a table that shows the probability distribution of x.

Let X = the net amount you gain on the single play of a game.

X	-\$1	\$2
P_i	0.75	0.25

2. Compute the <u>expected value</u> of X. Explain what this means for the player.

$$\mu_x = E(x) = \Sigma x_i p_i$$

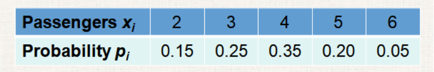
$$\mu = E(x) = -1(.75) + 2(.25)$$

$$= -.75 + .50$$

$$\boxed{= -\$0.25}$$

Notes - 6.2

Jeep problem... Passengers



Passengers x_i	2	3	4	5	6
Probability p_i	0.15	0.25	0.35	0.20	0.05

$$\boxed{\mu_x = E(x) = \Sigma x_i p_i}$$

Mean = (2 x .15) + (3 x .25) + (4 x .35) + (5 x .2) + (6 x .05)

= 3.75

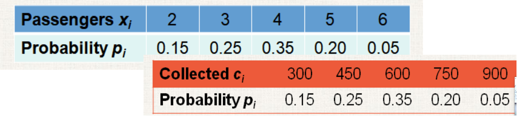
$$\boxed{Var(x) = \sigma_x^2 = (x_1 - \mu_x)^2 p_1 + (x_2 - \mu_x)^2 p_2 + ...}$$

Variance = $(2-3.75)^2(0.15) + (3-3.75)^2(0.25) + (4-3.75)^2(0.35) + (5-3.75)^2(0.20) + (6-3.75)^2(0.05)$

= 1.1875

Standard deviation $\sigma_x = \sqrt{1.1875} \approx 1.090$

Jeep problem... Gross Profit



Passengers x_i	2	3	4	5	6
Probability p_i	0.15	0.25	0.35	0.20	0.05

Collected c_i	300	450	600	750	900
Probability p_i	0.15	0.25	0.35	0.20	0.05

$$\boxed{\mu_x = E(x) = \Sigma x_i p_i}$$

Mean = (300 x .15) + (450 x .25) + (600 x .35) + (750 x .2) + (900 x .05)

= \$562.50

$$\boxed{Var(x) = \sigma_x^2 = (x_1 - \mu_x)^2 p_1 + (x_2 - \mu_x)^2 p_2 + ...}$$

Variance = $(300-562.50)^2(0.15) + (450-562.50)^2(0.25) + (600-562.50)^2(0.35) + (750-562.50)^2(0.20) + (900-562.50)^2(0.05)$

= 26,718.75

Standard deviation $\sigma_x = \sqrt{26{,}718.75} \approx \$163.46$

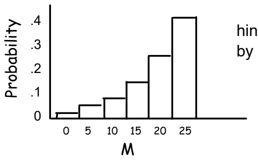
6.2 Combining random variables solutions

Page 382, #35,37,39-41

35. $\mu_y = 2.54(1.2) = 3.048$ cm

$\sigma_y = 2.54(0.25) = 0.635$ cm

37. a.



hint: multiply by $5

b. μ_M = $19.35. If many ferry tips were selected at random, the ferry would collect about $19.35 per trip, on average. (mean x $5)

c. σ_M = $6.45. The amounts collected on randomly selected ferry trips will typically VARY by about $6.45 from the mean of $19.35. (S.D. x 5)

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39. a. $\mu_G = 7.6(5) + 50 = 88$

b. $\sigma_G = 5(1.32) = 6.6$

c. $\sigma_G^2 = (5\sigma_x)^2 = 25\sigma_x^2$

The variance of G is 25 times the variance of X.

40. a. $\text{median}_G = 5(8.5) + 50 = 92.5$

b. $\text{IQR}_G = 5(9 - 8) = 5$

c. The shape will be the same is the distribution of X, because multiplying a constant and adding a constant doesn't change the shape.

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41. a. μ_y = -$0.65. If many ferry trips were selected at random, the ferry would lose about $0.65 per trip, on average.

X cars	0	1	2	3	4	5
	.02	.05	.08	.16	.27	.42

$\mu_x = 3.87$

$\sigma_x = 1.29$

M dollars	0	5	10	15	20	25
X x 5	.02	.05	.08	.16	.27	.42

Y profit	-20	-15	-10	-5	0	5
M - 20	.02	.05	.08	.16	.27	.42

work:

$\mu_M = 5(3.87) - 20 = \$19.35$ per trip earned

$20 cost... 19.35 - 20.00 = -$0.65

b. σ_y = $6.45

The amount of profit on randomly selected ferry trips will be about $6.45 from the mean of -$0.65.

work: $\sigma_y = \$1.29 \times 5 = \$6.45$

Use this information for problems 1-3: Choose an American Household at random and let the random variable X be the number of cars (including SUVs and light trucks) they own. Here is the probability model if we ignore the few households that own more than 5 cars. 0 Cars = 0.09, 1 Car = 0.36, 2 Cars = 0.35, 3 Cars = 0.13, 4 Cars = 0.05, and 5 Cars = 0.02.

What's the expected number of cars in a randomly selected American Household?

$$\mu_X = \Sigma (X_i P_i)$$

$\mu = 0(0.09) + 1(0.36) + 2(0.35) + 3(.13) + 4(0.05) + 5(0.02)$

$\mu_X = 1.75$

$\sigma_x^2 = \Sigma (x - \mu)^2 P_i$

$= (0 - 1.75)^2(.09) + (1 - 1.75)^2(.36) + (2 - 1.75)^2(.35)$
$+ (3 - 1.75)^2(.13) + (4 - 1.75)^2(.05) + (5 - 1.75)^2(.02)$

$\sigma_x^2 = 1.1675$

$\sigma_x = \sqrt{1.1675}$

$\sigma = 1.08$

Use this information for problems 1-3: Choose an American Household at random and let the random variable X be the number of cars (including SUVs and light trucks) they own. Here is the probability model if we ignore the few households that own more than 5 cars. 0 Cars = 0.09, 1 Car = 0.36, 2 Cars = 0.35, 3 Cars = 0.13, 4 Cars = 0.05, and 5 Cars = 0.02. 2. The Standard Deviation of X is X = 1.08. If many households were selected at random, which of the following would be the best interpretation of the value 1.08?

Mean of 1.75 cars

standard deviation of 1.08...

b. The number of cars would typically be about 1.08 from the mean.

A randomly selected household will have an amount of cars that will typically deviate 1.08 cars from 1.75 cars.

Choose an American Household at random and let the random variable X be the number of cars (including SUVs and light trucks) they own. Here is the probability model if we ignore the few households that own more than 5 cars. 0 Cars = 0.09, 1 Car = 0.36, 2 Cars = 0.35, 3 Cars = 0.13, 4 Cars = 0.05, and 5 Cars = 0.02.

About what percentage of households have a number of cars within 2 standard deviations of the mean?

95% of the data

93% of the data

Page 384, #45 and 47

$\mu = 8.5$ c
$\sigma = 2.25$ c

45. a. $\mu_y = 47.3$ degrees F

$\sigma_y = 4.05$ degrees F

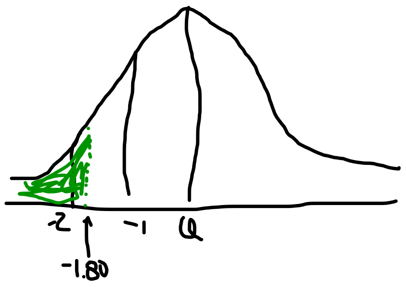
b. Y has the N(47.3,4.05) distribution.

We want to find $P(Y < 40)$.

$$z = \frac{(40-47.3)}{4.05} = -1.80$$

and $P(Z < -1.80) = 0.0359$.

There is a 0.0359 probability that the midnight temperature in the cabin is below 40 degrees.



47. a. yes, the mean of a sum is always equal to the sum of the means.

b. No, because it is not reasonable to assume that X and Y are independent.

Chapter 6.3 - Part 1: Binomial and Geometric Random Variables

To find the Binomial Coefficient...

Explained on Page 392...

On your graphing calculator!!!

"n choose r"

Keys: MATH .... PRB 3: nCr

5... math ... prb ... 3. nCr... 3

Mean x = 5(.25) = 1.25

Standard Deviation x

$\sqrt{5(.25)(1 - .25)}$ = 0.968

Homework! Page 384, #45, 47

45

Hints: a. Convert mean to degrees fahrenheit.

Standard Deviation conversion does NOT include adding 32 degrees. (why?)

b. Use the z score calculation and the standard normal table. Remember, we only want the probability that is it Less than 40 degrees fahrenheit.

47

Hints: review the rules for adding, subtracting and multiplying mean values as long as they are INDEPENDENT.

Hints: review the rules for adding subtracting and multiplying standard deviation as long as they are independent.

Homework Solutions - 6.3-1

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69. Binomial...

Binary? Success is seed germinates, failure is seed does not germinate.

Independent? Yes, one seed's outcome does not determine another's.

Number? n = 20

Success? p = 0.85

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71. Not Binomial

Binary? Success is person is lefty, failure is person is righty.

Independent? because they are random, their handedness is independent.

Number? No, the number of trials is not fixed.

Sucess? p = 0.10

75. $P(X = 4) = \left[{7 \atop 4}\right] (0.44)^4(0.56)^3 =$

$= 35\ (.03748...)(.175616) =$

$P(X=4) = 0.2304$

There is a .2304 probability that exactly 4 of the 7 elk survive to adulthood.

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77. $P(x<4) =$

$\left[{7 \atop 5}\right] (0.44)^5(0.56)^2 =$ .108607

$\left[{7 \atop 6}\right] (0.44)^6(0.56)^1 =$ .028444

$\left[{7 \atop 7}\right] (0.44)^7(0.56)^0 =$ .003193

$P(x<4) = .108607+.028444+.003193$

$= 0.1402$

Because this probability isn't very small, it is not surprising for more than 4 elk to survive to adulthood.

Bellwork! January 14th, 2021

Dead Batteries!

Suppose that your junk drawer has 8 AAA batteries but only 6 of them are good. You need to choose 4 for your graphing calculator. If you randomly select 4 batteries, what is the probability that all 4 of them work?

Don't do the calculation yet..

There's something wrong...

Thoughts???

$(6/8) \times (5/7) \times (4/6) \times (3/5) =$

0.214

Probability that a battery is good.

P(X is good) = .75

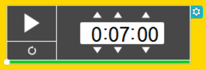
P(X is bad) = .25

8 choose 4

$P(x = 4) = \binom{8}{4} (.74)^4(.25)^4$

70 (.299986) (.003906)

Welcome! January 13th, 2021



FAST FOOD!!!

A fast food restaurant runs a promotion in which certain food items come with game pieces. According to the restaurant, 1 in 4 game pieces is a winner.

If you get 4 game pieces, what is the probability that you win exactly 1 prize?

0.422

0.1054

0.105

BINOMIAL PROBABILITY

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$P(x=1) = \binom{4}{1} (.25)^1 (.75)^3$$

YNNN NYNN NNYN NNNY

P(X = 1)

for 1 game piece

Probability of success is .25

Probability of failure is .75

$P(x = 1) = \binom{4}{1}(.25)^1(.75)^3$

P(x = 1) = .4219

$P(x \leq 2) =$

$\binom{4}{1}(.25)^1(.75)^3 + \binom{4}{2}(.25)^2(.75)^2$

.4219 + .2109 = .6328

$P(x = 0)\ \binom{4}{0}(.25)^0(.75)^4$

P(x= 0) = .3164

$P(x = 0) = \binom{4}{0}(.25)^0(.75)^4$ .316406

$P(x = 1) = \binom{4}{1}(.25)^1(.75)^3$ .421875

$P(x = 2) = \binom{4}{2}(.25)^2(.75)^2$ .2109375

$P(x = 3) = \binom{4}{3}(.25)^3(.75)^1$ .046875

$P(x = 4) = \binom{4}{4}(.25)^4(.75)^0$ .003906

X	0	1	2	3	4
Px	.3164	.4219	.2109	.0469	.0039

$P(x \leq 2) =$

$\binom{4}{1}(.25)^1(.75)^3 + \binom{4}{2}(.25)^2(.75)^2$

.4219 + .2109 = .6328

$P(x = 0)\ \binom{4}{0}(.25)^0(.75)^4$

P(x= 0) = .3164

$P(X < 4)$

P(0.25)

X = 4

upper limit = 3

Binomial CDF (better way)

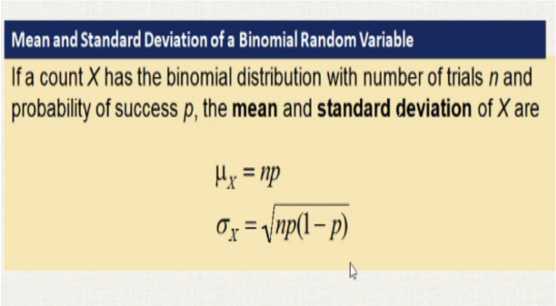
"cumulative Distribution function)

$\boxed{\text{2nd}}\boxed{\text{VARS}}$ - A: binomcdf(

(Trials, Prob, upper bound X)

(4, .25, 3)

X = 0.996093



Mean and Standard Deviation of a Binomial Random Variable

If a count X has the binomial distribution with number of trials n and probability of success p, the **mean** and **standard deviation** of X are

$$\mu_X = np$$

$$\sigma_X = \sqrt{np(1-p)}$$

6.3 Binomial Random Variables continued

#79, 81, 83

79. a. $P(X = 17) = \binom{20}{17}(.85)^{17}(.15)^{3} = 0.2428$

b. $P(X<12) = \binom{20}{0}(.85)^{0}(.15)^{20} + \ldots \binom{20}{12}(.85)^{12}(.15)^{8}$

= 0.0059. This is low. Judy should be suspicious.

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81. a. $\mu_x = 15(0.20) = 3$

If we watched the machine make sets of 15 calls, we would expect about 3 calls to reach a live person, on average.

b. $\sigma_x = \sqrt{15(0.20)(0.80)}$

= 1.55. If we watched the machine make many sets of 15 calls, we would expect the number of calls that reach a live person to typically vary by about 1.55 from the mean of 3.

83. a. $\mu_y = 15(0.80) = 12$

Notice that $\mu_x = 3$ ... and 12 + 3 = 15 (total # of calls)

b. $\sigma_y = \sqrt{15(0.80)(0.20)}$

= 1.55

This is the same value as σ_x because $Y = 15 - X$ and adding a constant to a random variable doesn't change the spread.