

## CHAPTER 3

## Mid-Chapter Check Point

**WHAT YOU KNOW:** We graphed quadratic functions using vertices, intercepts, and additional points, as necessary. We learned that the vertex of  $f(x) = a(x - h)^2 + k$  is  $(h, k)$  and the vertex of  $f(x) = ax^2 + bx + c$  is  $(-\frac{b}{2a}, f(-\frac{b}{2a}))$ . We used the vertex to solve problems that involved minimizing or maximizing quadratic functions. We learned a number of techniques for finding the zeros of a polynomial function  $f$  of degree 3 or higher or, equivalently, finding the roots, or solutions, of the equation  $f(x) = 0$ . For some functions, the zeros were found by factoring  $f(x)$ . For other functions, we listed possible rational zeros and used synthetic division and the Factor Theorem to determine the zeros. We saw that graphs cross the  $x$ -axis at zeros of odd multiplicity and touch the  $x$ -axis and turn around at zeros of even multiplicity. We learned to graph polynomial functions using zeros, the Leading Coefficient Test, intercepts, and symmetry. We checked graphs using the fact that a polynomial function of degree  $n$  has a graph with at most  $n - 1$  turning points. After finding zeros of polynomial functions, we reversed directions by using the Linear Factorization Theorem to find functions with given zeros.

In Exercises 1–4, graph the given quadratic function. Give each function's domain and range.

- $f(x) = (x - 3)^2 - 4$
- $f(x) = 5 - (x + 2)^2$
- $f(x) = -x^2 - 4x + 5$
- $f(x) = 3x^2 - 6x + 1$

In Exercises 5–13, find all zeros of each polynomial function. Then graph the function.

- $f(x) = (x - 2)^2(x + 1)^3$
- $f(x) = -(x - 2)^2(x + 1)^2$
- $f(x) = x^3 - x^2 - 4x + 4$
- $f(x) = x^4 - 5x^2 + 4$
- $f(x) = -(x + 1)^6$
- $f(x) = -6x^3 + 7x^2 - 1$

- $f(x) = 2x^3 - 2x$
- $f(x) = x^3 - 2x^2 + 26x$
- $f(x) = -x^3 + 5x^2 - 5x - 3$

In Exercises 14–19, solve each polynomial equation.

- $x^3 - 3x + 2 = 0$
- $6x^3 - 11x^2 + 6x - 1 = 0$
- $(2x + 1)(3x - 2)^3(2x - 7) = 0$
- $2x^3 + 5x^2 - 200x - 500 = 0$
- $x^4 - x^3 - 11x^2 = x + 12$
- $2x^4 + x^3 - 17x^2 - 4x + 6 = 0$
- A company manufactures and sells bath cabinets. The function

$$P(x) = -x^2 + 150x - 4425$$

models the company's daily profit,  $P(x)$ , when  $x$  cabinets are manufactured and sold per day. How many cabinets should be manufactured and sold per day to maximize the company's profit? What is the maximum daily profit?

- Among all pairs of numbers whose sum is  $-18$ , find a pair whose product is as large as possible. What is the maximum product?
- The base of a triangle measures 40 inches minus twice the measure of its height. For what measure of the height does the triangle have a maximum area? What is the maximum area?

In Exercises 23–24, divide, using synthetic division if possible.

- $(6x^4 - 3x^3 - 11x^2 + 2x + 4) \div (3x^2 - 1)$
- $(2x^4 - 13x^3 + 17x^2 + 18x - 24) \div (x - 4)$

In Exercises 25–26, find an  $n$ th-degree polynomial function with real coefficients satisfying the given conditions.

- $n = 3$ ; 1 and  $i$  are zeros;  $f(-1) = 8$
- $n = 4$ ; 2 (with multiplicity 2) and  $3i$  are zeros;  $f(0) = 36$
- Does  $f(x) = x^3 - x - 5$  have a real zero between 1 and 2?