CHAPTER 4 Mid-Chapter Check Point

WHAT YOU KNOW: We evaluated and graphed exponential functions $[f(x) = b^x, b > 0$ and $b \neq 1]$, including the natural exponential function $[f(x) = e^x$, $e \approx 2.718]$. A function has an inverse that is a function if there is no horizontal line that intersects the function's graph more than once. The exponential function passes this horizontal line test and we called the inverse of the exponential function with base b the logarithmic function with base b. We learned that $y = \log_b x$ is equivalent to $b^y = x$. We evaluated and graphed logarithmic functions, including the common logarithmic function $[f(x) = \log_{10} x$ or $f(x) = \log x]$ and the natural logarithmic function $[f(x) = \log_e x$ or $f(x) = \ln x]$. We learned to use transformations to graph exponential and logarithmic functions. Finally, we used properties of logarithms to expand and condense logarithmic expressions.

In Exercises 1–5, graph f and g in the same rectangular coordinate system. Graph and give equations of all asymptotes. Give each function's domain and range.

1. $f(x) = 2^x$ and $g(x) = 2^x - 3$
2. $f(x) = \left(\frac{1}{2}\right)^x$ and $g(x) = \left(\frac{1}{2}\right)^{x-1}$
3. $f(x) = e^x$ and $g(x) = \ln x$
4. $f(x) = \log_2 x$ and $g(x) = \log_2(x - 1) + 1$
5. $f(x) = \log_{\frac{1}{2}} x$ and $g(x) = -2 \log_{\frac{1}{2}} x$

In Exercises 6–9, find the domain of each function.

6. $f(x) = \log_3(x + 6)$
7. $g(x) = \log_3 x + 6$
8. $h(x) = \log_3(x + 6)^2$
9. $f(x) = 3^{x+6}$

In Exercises 10–20, evaluate each expression without using a calculator. If evaluation is not possible, state the reason.

10. $\log_2 8 + \log_5 25$
11. $\log_3 \frac{1}{9}$
12. $\log_{100} 10$
13. $\log \sqrt[3]{10}$
14. $\log_2(\log_3 81)$
15. $\log_3\left(\log_2 \frac{1}{8}\right)$
16. $6^{\log_6 5}$
17. $\ln e^{\sqrt{7}}$
18. $10^{\log 13}$
19. $\log_{100} 0.1$
20. $\log_\pi \pi^{\sqrt{\pi}}$

In Exercises 21–22, expand and evaluate numerical terms.

21. $\log\left(\frac{\sqrt{xy}}{1000}\right)$
22. $\ln(e^{19}x^{20})$

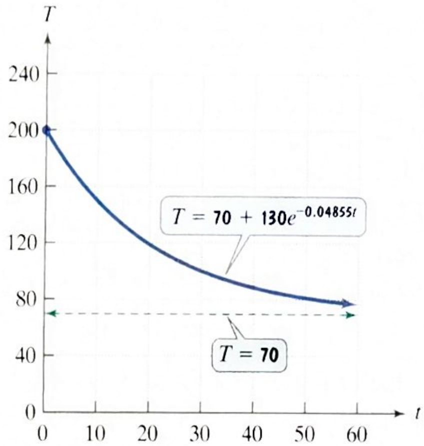
In Exercises 23–25, write each expression as a single logarithm.

23. $8 \log_7 x - \frac{1}{3}\log_7 y$
24. $7 \log_5 x + 2 \log_5 x$
25. $\frac{1}{2}\ln x - 3 \ln y - \ln(z - 2)$
26. Use the formulas

$$A = P\left(1 + \frac{r}{n}\right)^{nt} \quad \text{and} \quad A = Pe^{rt}$$

to solve this exercise. You decide to invest \$8000 for 3 years at an annual rate of 8%. How much more is the return if the interest is compounded continuously than if it is compounded monthly? Round to the nearest dollar.

12. A cup of coffee is taken out of a microwave oven and placed in a room. The temperature, T, in degrees Fahrenheit, of the coffee after t minutes is modeled by the function $T = 70 + 130e^{-0.04855t}$. The graph of the function is shown in the figure.



Use the graph to answer each of the following questions.

a. What was the temperature of the coffee when it was first taken out of the microwave?

b. What is a reasonable estimate of the temperature of the coffee after 20 minutes? Use your calculator to verify this estimate.

c. What is the limit of the temperature to which the coffee will cool? What does this tell you about the temperature of the room?

4.2

In Exercises 13–15, write each equation in its equivalent exponential form.

13. $\frac{1}{2} = \log_{49} 7$ **14.** $3 = \log_4 x$ **15.** $\log_3 81 = y$

In Exercises 16–18, write each equation in its equivalent logarithmic form.

16. $6^3 = 216$ **17.** $b^4 = 625$ **18.** $13^y = 874$

In Exercises 19–29, evaluate each expression without using a calculator. If evaluation is not possible, state the reason.

19. $\log_4 64$ **20.** $\log_5 \frac{1}{25}$ **21.** $\log_3(-9)$

22. $\log_{16} 4$ **23.** $\log_{17} 17$ **24.** $\log_3 3^8$

25. $\ln e^5$ **26.** $\log_3 \frac{1}{\sqrt{3}}$ **27.** $\ln \frac{1}{e^2}$

28. $\log \frac{1}{1000}$ **29.** $\log_3(\log_8 8)$

30. Graph $f(x) = 2^x$ and $g(x) = \log_2 x$ in the same rectangular coordinate system. Use the graphs to determine each function's domain and range.

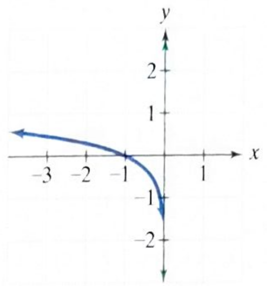
31. Graph $f(x) = \left(\frac{1}{3}\right)^x$ and $g(x) = \log_{\frac{1}{3}} x$ in the same rectangular coordinate system. Use the graphs to determine each function's domain and range.

In Exercises 32–35, the graph of a logarithmic function is given. Select the function for each graph from the following options:

$f(x) = \log x$, $g(x) = \log(-x)$,

$h(x) = \log(2 - x)$, $r(x) = 1 + \log(2 - x)$.

32.



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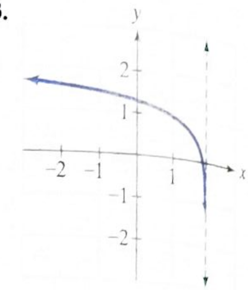

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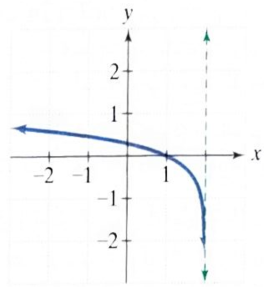

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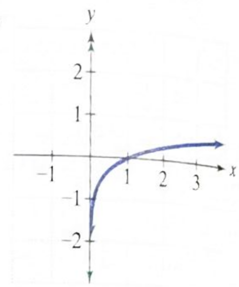


In Exercises 36–38, begin by graphing $f(x) = \log_2 x$. Then use transformations of this graph to graph the given function. What is the graph's x-intercept? What is the vertical asymptote? Use the graphs to determine each function's domain and range.

36. $g(x) = \log_2(x - 2)$ **37.** $h(x) = -1 + \log_2 x$

38. $r(x) = \log_2(-x)$

In Exercises 39–40, graph f and g in the same rectangular coordinate system. Use transformations of the graph of f to obtain the graph of g. Graph and give equations of all asymptotes. Use the graphs to determine each function's domain and range.

39. $f(x) = \log x$ and $g(x) = -\log(x + 3)$

40. $f(x) = \ln x$ and $g(x) = -\ln(2x)$

In Exercises 41–43, find the domain of each logarithmic function.

41. $f(x) = \log_8(x + 5)$ **42.** $f(x) = \log(3 - x)$

43. $f(x) = \ln(x - 1)^2$

In Exercises 44–46, use inverse properties of logarithms to simplify each expression.

44. $\ln e^{6x}$ **45.** $e^{\ln\sqrt{x}}$ **46.** $10^{\log 4x^2}$

47. On the Richter scale, the magnitude, R, of an earthquake of intensity I is given by $R = \log \frac{I}{I_0}$, where I_0 is the intensity of a barely felt zero-level earthquake. If the intensity of an earthquake is $1000I_0$, what is its magnitude on the Richter scale?

48. Students in a psychology class took a final examination. As part of an experiment to see how much of the course content they remembered over time, they took equivalent forms of the exam in monthly intervals thereafter. The average score, $f(t)$, for the group after t months is modeled by the function $f(t) = 76 - 18\log(t + 1)$, where $0 \le t \le 12$.

a. What was the average score when the exam was first given?

b. What was the average score after 2 months? 4 months? 6 months? 8 months? one year?

c. Use the results from parts (a) and (b) to graph f. Describe what the shape of the graph indicates in terms of the material retained by the students.

49. The formula

$$t = \frac{1}{c}\ln\left(\frac{A}{A-N}\right)$$

describes the time, t, in weeks, that it takes to achieve mastery of a portion of a task. In the formula, A represents maximum learning possible, N is the portion of the learning that is to be achieved, and c is a constant used to measure an individual's learning style. A 50-year-old man decides to start running as a way to maintain good health. He feels that the maximum rate he could ever hope to achieve is 12 miles per hour. How many weeks will it take before the man can run 5 miles per hour if $c = 0.06$ for this person?

4.3

In Exercises 50–53, use properties of logarithms to expand each logarithmic expression as much as possible. Where possible, evaluate logarithmic expressions without using a calculator.

50. $\log_6(36x^3)$

51. $\log_4\left(\frac{\sqrt{x}}{64}\right)$

52. $\log_2\left(\frac{xy^2}{64}\right)$

53. $\ln\sqrt[3]{\frac{x}{e}}$

In Exercises 54–57, use properties of logarithms to condense each logarithmic expression. Write the expression as a single logarithm whose coefficient is 1.

54. $\log_b 7 + \log_b 3$

55. $\log 3 - 3\log x$

56. $3\ln x + 4\ln y$

57. $\frac{1}{2}\ln x - \ln y$

In Exercises 58–59, use common logarithms or natural logarithms and a calculator to evaluate to four decimal places.

58. $\log_6 72{,}348$

59. $\log_4 0.863$

In Exercises 60–63, determine whether each equation is true or false. Where possible, show work to support your conclusion. If the statement is false, make the necessary change(s) to produce a true statement.

60. $(\ln x)(\ln 1) = 0$

61. $\log(x+9) - \log(x+1) = \frac{\log(x+9)}{\log(x+1)}$

62. $(\log_2 x)^4 = 4\log_2 x$

63. $\ln e^x = x\ln e$

4.4

In Exercises 64–73, solve each exponential equation. Where necessary, express the solution set in terms of natural or common logarithms and use a calculator to obtain a decimal approximation, correct to two decimal places, for the solution.

64. $2^{4x-2} = 64$

65. $125^x = 25$

66. $10^x = 7000$

67. $9^{x+2} = 27^{-x}$

68. $8^x = 12{,}143$

69. $9e^{5x} = 1269$

70. $e^{12-5x} - 7 = 123$

71. $5^{4x+2} = 37{,}500$

72. $3^{x+4} = 7^{2x-1}$

73. $e^{2x} - e^x - 6 = 0$

In Exercises 74–79, solve each logarithmic equation.

74. $\log_4(3x-5) = 3$

75. $3 + 4\ln(2x) = 15$

76. $\log_2(x+3) + \log_2(x-3) = 4$

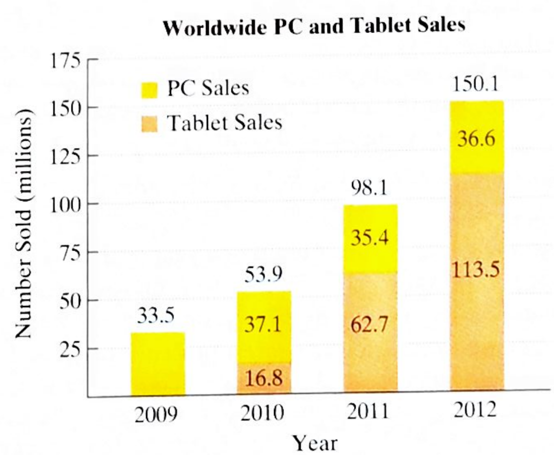
77. $\log_3(x-1) - \log_3(x+2) = 2$

78. $\ln(x+4) - \ln(x+1) = \ln x$

79. $\log_4(2x+1) = \log_4(x-3) + \log_4(x+5)$

80. The function $P(x) = 14.7e^{-0.21x}$ models the average atmospheric pressure, $P(x)$, in pounds per square inch, at an altitude of x miles above sea level. The atmospheric pressure at the peak of Mt. Everest, the world's highest mountain, is 4.6 pounds per square inch. How many miles above sea level, to the nearest tenth of a mile, is the peak of Mt. Everest?

81. **Newest Dinosaur: The PC-osaurus?** For the period from 2009 through 2012, worldwide PC sales stayed relatively flat, while tablet sales skyrocketed. The bar graph shows worldwide PC and tablet sales, in millions, from 2009 through 2012.



Source: Canalys

The function

$$f(t) = 33.4(1.66)^t$$

models worldwide PC and tablet sales combined, $f(t)$, in millions, t years after 2009. When does this model project that 421 million PC and tablets were sold? Round to the nearest year. Based on the relatively flat PC sales over the four years shown by the graph, estimate the number of tablet sales for that year.