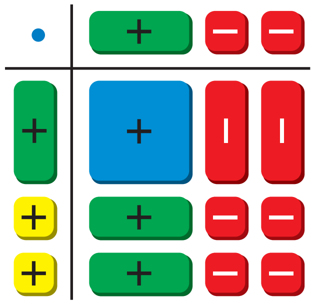
7.3 Special Products of Polynomials

Essential Question What are the patterns in the special products $(a + b)(a - b)$, $(a + b)^2$, and $(a - b)^2$?

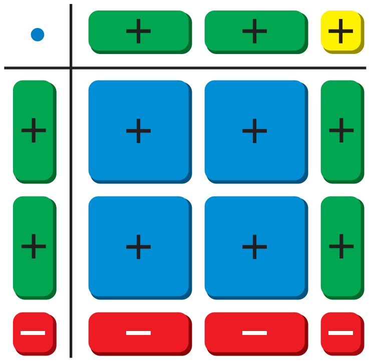
EXPLORATION 1 Finding a Sum and Difference Pattern

Work with a partner. Write the product of two binomials modeled by each rectangular array of algebra tiles.

a. $(x + 2)(x - 2) =$



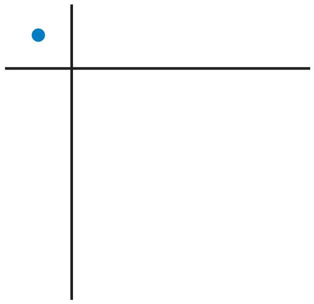
b. $(2x - 1)(2x + 1) =$



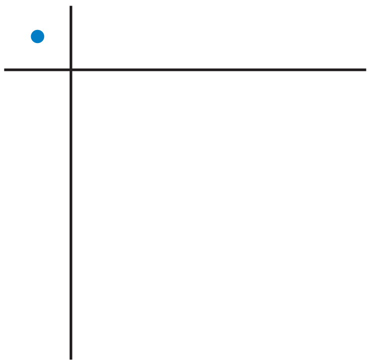
EXPLORATION 2 Finding the Square of a Binomial Pattern

Work with a partner. Draw the rectangular array of algebra tiles that models each product of two binomials. Write the product.

a. $(x + 2)^2 =$



b. $(2x - 1)^2 =$



LOOKING FOR STRUCTURE

To be proficient in math, you need to look closely to discern a pattern or structure.

Communicate Your Answer

3. What are the patterns in the special products $(a + b)(a - b)$, $(a + b)^2$, and $(a - b)^2$?

4. Use the appropriate special product pattern to find each product. Check your answers using algebra tiles.

a. $(x + 3)(x - 3)$ **b.** $(x - 4)(x + 4)$ **c.** $(3x + 1)(3x - 1)$

d. $(x + 3)^2$ **e.** $(x - 2)^2$ **f.** $(3x + 1)^2$

7.3 Lesson

Core Vocabulary

Previous
binomial

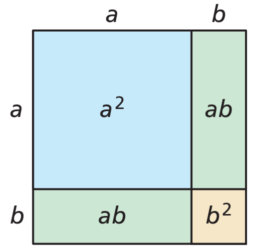
What You Will Learn

- Use the square of a binomial pattern.
- Use the sum and difference pattern.
- Use special product patterns to solve real-life problems.

Using the Square of a Binomial Pattern

The diagram shows a square with a side length of $(a + b)$ units. You can see that the area of the square is

$$(a + b)^2 = a^2 + 2ab + b^2.$$



This is one version of a pattern called the square of a binomial. To find another version of this pattern, use algebra: replace b with $-b$.

$(a + (-b))^2 = a^2 + 2a(-b) + (-b)^2$ — Replace b with $-b$ in the pattern above.

$(a - b)^2 = a^2 - 2ab + b^2$ — Simplify.

Core Concept

Square of a Binomial Pattern

Algebra	Example
$(a + b)^2 = a^2 + 2ab + b^2$	$(x + 5)^2 = (x)^2 + 2(x)(5) + (5)^2$ $= x^2 + 10x + 25$
$(a - b)^2 = a^2 - 2ab + b^2$	$(2x - 3)^2 = (2x)^2 - 2(2x)(3) + (3)^2$ $= 4x^2 - 12x + 9$

LOOKING FOR STRUCTURE

When you use special product patterns, remember that a and b can be numbers, variables, or variable expressions.

EXAMPLE 1 Using the Square of a Binomial Pattern

Find each product.

a. $(3x + 4)^2$ **b.** $(5x - 2y)^2$

SOLUTION

a. $(3x + 4)^2 = (3x)^2 + 2(3x)(4) + 4^2$ — Square of a binomial pattern

$= 9x^2 + 24x + 16$ — Simplify.

- The product is $9x^2 + 24x + 16$.

b. $(5x - 2y)^2 = (5x)^2 - 2(5x)(2y) + (2y)^2$ — Square of a binomial pattern

$= 25x^2 - 20xy + 4y^2$ — Simplify.

- The product is $25x^2 - 20xy + 4y^2$.

Monitoring Progress

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Find the product.

1. $(x + 7)^2$ **2.** $(7x - 3)^2$ **3.** $(4x - y)^2$ **4.** $(3m + n)^2$

Using the Sum and Difference Pattern

To find the product $(x + 2)(x - 2)$, you can multiply the two binomials using the FOIL Method.

$$
\begin{aligned}
(x + 2)(x - 2) &= x^2 - 2x + 2x - 4 && \text{FOIL Method} \\
&= x^2 - 4 && \text{Combine like terms.}
\end{aligned}
$$

This suggests a pattern for the product of the sum and difference of two terms.

Core Concept

Sum and Difference Pattern

Algebra	**Example**
$(a + b)(a - b) = a^2 - b^2$	$(x + 3)(x - 3) = x^2 - 9$

EXAMPLE 2 Using the Sum and Difference Pattern

Find each product.

a. $(t + 5)(t - 5)$

b. $(3x + y)(3x - y)$

SOLUTION

a.
$$
\begin{aligned}
(t + 5)(t - 5) &= t^2 - 5^2 && \text{Sum and difference pattern} \\
&= t^2 - 25 && \text{Simplify.}
\end{aligned}
$$

▶ The product is $t^2 - 25$.

b.
$$
\begin{aligned}
(3x + y)(3x - y) &= (3x)^2 - y^2 && \text{Sum and difference pattern} \\
&= 9x^2 - y^2 && \text{Simplify.}
\end{aligned}
$$

▶ The product is $9x^2 - y^2$.

The special product patterns can help you use mental math to find certain products of numbers.

EXAMPLE 3 Using Special Product Patterns and Mental Math

Use special product patterns to find the product $26 \cdot 34$.

SOLUTION

Notice that 26 is 4 less than 30, while 34 is 4 more than 30.

$$
\begin{aligned}
26 \cdot 34 &= (30 - 4)(30 + 4) && \text{Write as product of difference and sum.} \\
&= 30^2 - 4^2 && \text{Sum and difference pattern} \\
&= 900 - 16 && \text{Evaluate powers.} \\
&= 884 && \text{Simplify.}
\end{aligned}
$$

▶ The product is 884.

Monitoring Progress

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Find the product.

5. $(x + 10)(x - 10)$

6. $(2x + 1)(2x - 1)$

7. $(x + 3y)(x - 3y)$

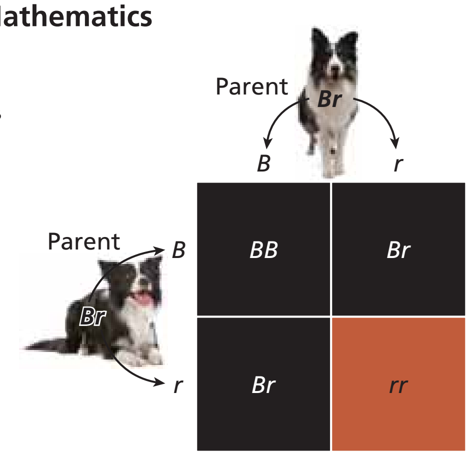
8. Describe how to use special product patterns to find 21^2.

Solving Real-Life Problems

EXAMPLE 4 Modeling with Mathematics

A combination of two genes determines the color of the dark patches of a border collie's coat. An offspring inherits one patch color gene from each parent. Each parent has two color genes, and the offspring has an equal chance of inheriting either one.

The gene B is for black patches, and the gene r is for red patches. Any gene combination with a B results in black patches. Suppose each parent has the same gene combination Br. The Punnett square shows the possible gene combinations of the offspring and the resulting patch colors.



a. What percent of the possible gene combinations result in black patches?

b. Show how you could use a polynomial to model the possible gene combinations.

SOLUTION

a. Notice that the Punnett square shows four possible gene combinations of the offspring. Of these combinations, three result in black patches.

▶ So, 75% of the possible gene combinations result in black patches.

b. Model the gene from each parent with $0.5B + 0.5r$. There is an equal chance that the offspring inherits a black or a red gene from each parent.

You can model the possible gene combinations of the offspring with $(0.5B + 0.5r)^2$. Notice that this product also represents the area of the Punnett square.

Expand the product to find the possible patch colors of the offspring.

$$(0.5B + 0.5r)^2 = (0.5B)^2 + 2(0.5B)(0.5r) + (0.5r)^2$$

$$= 0.25B^2 + 0.5Br + 0.25r^2$$

Consider the coefficients in the polynomial.

$$0.25B^2 + 0.5Br + 0.25r^2$$

25% BB, black patches

50% Br, black patches

25% rr, red patches

The coefficients show that 25% + 50% = 75% of the possible gene combinations result in black patches.

Monitoring Progress

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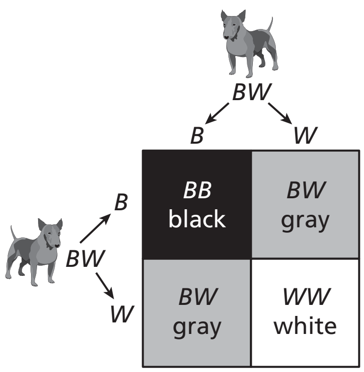


9. Each of two dogs has one black gene (B) and one white gene (W). The Punnett square shows the possible gene combinations of an offspring and the resulting colors.

a. What percent of the possible gene combinations result in black?

b. Show how you could use a polynomial to model the possible gene combinations of the offspring.

7.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** Explain how to use the square of a binomial pattern.

2. **WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

$(x+1)(x-1)$	$(3x+2)(3x-2)$	$(x+2)(x-3)$	$(2x+5)(2x-5)$

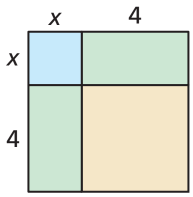
Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, find the product. *(See Example 1.)*

3. $(x+8)^2$
4. $(a-6)^2$
5. $(2f-1)^2$
6. $(5p+2)^2$
7. $(-7t+4)^2$
8. $(-12-n)^2$
9. $(2a+b)^2$
10. $(6x-3y)^2$

MATHEMATICAL CONNECTIONS **In Exercises 11–14, write a polynomial that represents the area of the square.**

11.



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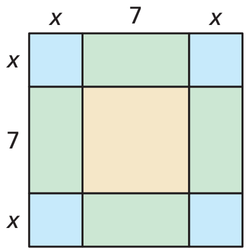

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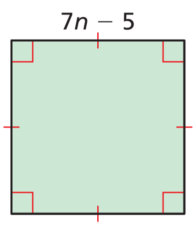

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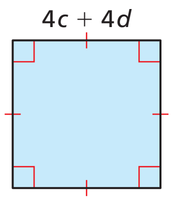


In Exercises 15–24, find the product. *(See Example 2.)*

15. $(t-7)(t+7)$
16. $(m+6)(m-6)$
17. $(4x+1)(4x-1)$
18. $(2k-4)(2k+4)$
19. $(8+3a)(8-3a)$
20. $\left(\frac{1}{2}-c\right)\left(\frac{1}{2}+c\right)$
21. $(p-10q)(p+10q)$
22. $(7m+8n)(7m-8n)$
23. $(-y+4)(-y-4)$
24. $(-5g-2h)(-5g+2h)$

In Exercises 25–30, use special product patterns to find the product. *(See Example 3.)*

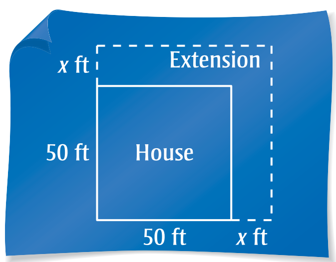
25. $16 \cdot 24$
26. $33 \cdot 27$
27. 42^2
28. 29^2
29. 30.5^2
30. $10\frac{1}{3} \cdot 9\frac{2}{3}$

ERROR ANALYSIS **In Exercises 31 and 32, describe and correct the error in finding the product.**

31. ✗
$$\begin{aligned}(k+4)^2 &= k^2 + 4^2 \\ &= k^2 + 16\end{aligned}$$

32. ✗
$$\begin{aligned}(s+5)(s-5) &= s^2 + 2(s)(5) - 5^2 \\ &= s^2 + 10s - 25\end{aligned}$$

33. **MODELING WITH MATHEMATICS** A contractor extends a house on two sides.



a. The area of the house after the renovation is represented by $(x+50)^2$. Find this product.

b. Use the polynomial in part (a) to find the area when $x = 15$. What is the area of the extension?

34. MODELING WITH MATHEMATICS A square-shaped parking lot with 100-foot sides is reduced by x feet on one side and extended by x feet on an adjacent side.

a. The area of the new parking lot is represented by $(100 - x)(100 + x)$. Find this product.

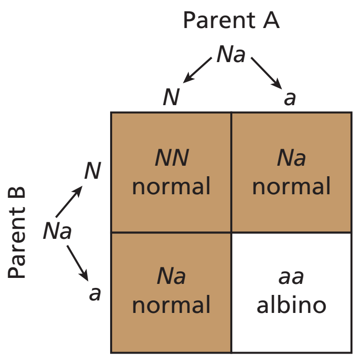
b. Does the area of the parking lot increase, decrease, or stay the same? Explain.

c. Use the polynomial in part (a) to find the area of the new parking lot when $x = 21$.

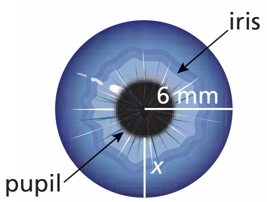
35. MODELING WITH MATHEMATICS In deer, the gene N is for normal coloring and the gene a is for no coloring, or albino. Any gene combination with an N results in normal coloring. The Punnett square shows the possible gene combinations of an offspring and the resulting colors from parents that both have the gene combination Na. *(See Example 4.)*

a. What percent of the possible gene combinations result in albino coloring?

b. Show how you could use a polynomial to model the possible gene combinations of the offspring.



36. MODELING WITH MATHEMATICS Your iris controls the amount of light that enters your eye by changing the size of your pupil.

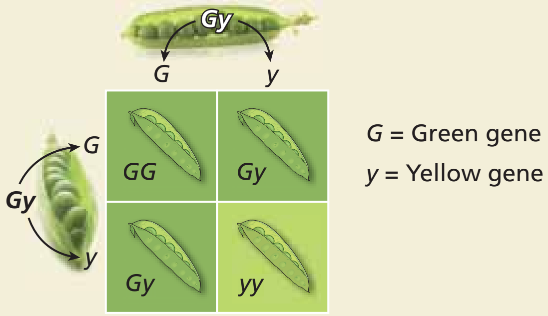


a. Write a polynomial that represents the area of your pupil. Write your answer in terms of π.

b. The width x of your iris decreases from 4 millimeters to 2 millimeters when you enter a dark room. How many times greater is the area of your pupil after entering the room than before entering the room? Explain.

37. CRITICAL THINKING Write two binomials that have the product $x^2 - 121$. Explain.

38. HOW DO YOU SEE IT? In pea plants, any gene combination with a green gene (G) results in a green pod. The Punnett square shows the possible gene combinations of the offspring of two Gy pea plants and the resulting pod colors.



A polynomial that models the possible gene combinations of the offspring is

$$(0.5G + 0.5y)^2 = 0.25G^2 + 0.5Gy + 0.25y^2.$$

Describe two ways to determine the percent of possible gene combinations that result in green pods.

In Exercises 39–42, find the product.

39. $(x^2 + 1)(x^2 - 1)$

40. $(y^3 + 4)^2$

41. $(2m^2 - 5n^2)^2$

42. $(r^3 - 6t^4)(r^3 + 6t^4)$

43. MAKING AN ARGUMENT Your friend claims to be able to use a special product pattern to determine that $\left(4\frac{1}{3}\right)^2$ is equal to $16\frac{1}{9}$. Is your friend correct? Explain.

44. THOUGHT PROVOKING The area (in square meters) of the surface of an artificial lake is represented by x^2. Describe three ways to modify the dimensions of the lake so that the new area can be represented by the three types of special product patterns discussed in this section.

45. REASONING Find k so that $9x^2 - 48x + k$ is the square of a binomial.

46. REPEATED REASONING Find $(x + 1)^3$ and $(x + 2)^3$. Find a pattern in the terms and use it to write a pattern for the cube of a binomial $(a + b)^3$.

47. PROBLEM SOLVING Find two numbers a and b such that $(a + b)(a - b) < (a - b)^2 < (a + b)^2$.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Factor the expression using the GCF. *(Skills Review Handbook)*

48. $12y - 18$

49. $9r + 27$

50. $49s + 35t$

51. $15x - 10y$