# 5.4 Solving Special Systems of Linear Equations

## Essential Question

Can a system of linear equations have no solution or infinitely many solutions?

### EXPLORATION 1 Using a Table to Solve a System

**Work with a partner.** You invest $450 for equipment to make skateboards. The materials for each skateboard cost $20. You sell each skateboard for $20.

**a.** Write the cost and revenue equations. Then copy and complete the table for your cost $C$ and your revenue $R$.

| $x$ (skateboards) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $C$ (dollars) | | | | | | | | | | | |
| $R$ (dollars) | | | | | | | | | | | |

**b.** When will your company break even? What is wrong?

**MODELING WITH MATHEMATICS**

To be proficient in math, you need to interpret mathematical results in real-life contexts.

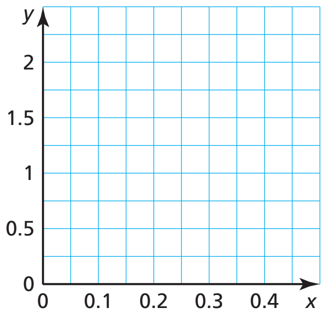
### EXPLORATION 2 Writing and Analyzing a System

**Work with a partner.** A necklace and matching bracelet have two types of beads. The necklace has 40 small beads and 6 large beads and weighs 10 grams. The bracelet has 20 small beads and 3 large beads and weighs 5 grams. The threads holding the beads have no significant weight.

**a.** Write a system of linear equations that represents the situation. Let $x$ be the weight (in grams) of a small bead and let $y$ be the weight (in grams) of a large bead.

**b.** Graph the system in the coordinate plane shown. What do you notice about the two lines?

**c.** Can you find the weight of each type of bead? Explain your reasoning.

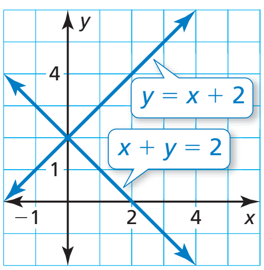



## Communicate Your Answer

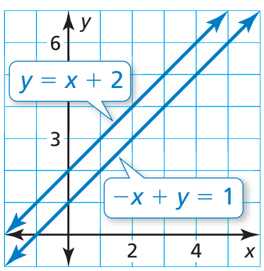
**3.** Can a system of linear equations have no solution or infinitely many solutions? Give examples to support your answers.

**4.** Does the system of linear equations represented by each graph have *no solution*, *one solution*, or *infinitely many solutions*? Explain.

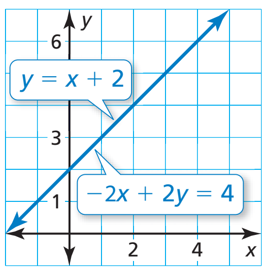
**a.**




**b.**




**c.**



# 5.4 Lesson

## What You Will Learn

- Determine the numbers of solutions of linear systems.
- Use linear systems to solve real-life problems.

### Core Vocabulary

***Previous***
parallel

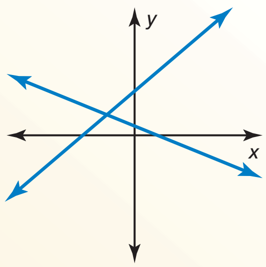
## The Numbers of Solutions of Linear Systems

### Core Concept

**Solutions of Systems of Linear Equations**

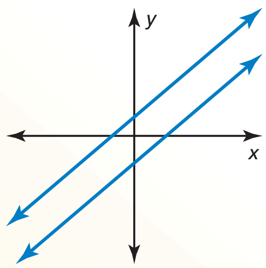
A system of linear equations can have *one solution*, *no solution*, or *infinitely many solutions*.

**One solution**



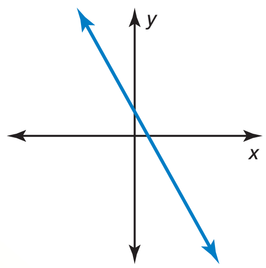

The lines intersect.

**No solution**




The lines are parallel.

**Infinitely many solutions**




The lines are the same.

### ANOTHER WAY

You can solve some linear systems by inspection. In Example 1, notice you can rewrite the system as

$-2x + y = 1$
$-2x + y = -5.$

This system has no solution because $-2x + y$ cannot be equal to both 1 and $-5$.

### EXAMPLE 1 Solving a System: No Solution

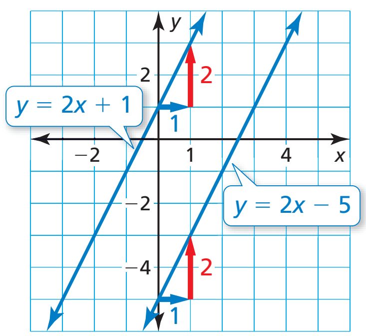
Solve the system of linear equations.

$y = 2x + 1$ Equation 1
$y = 2x - 5$ Equation 2

**SOLUTION**

**Method 1** Solve by graphing.

Graph each equation.




The lines have the same slope and different *y*-intercepts. So, the lines are parallel.

Because parallel lines do not intersect, there is no point that is a solution of both equations.

▶ So, the system of linear equations has no solution.

**Method 2** Solve by substitution.

Substitute $2x - 5$ for $y$ in Equation 1.

$y = 2x + 1$ Equation 1
$2x - 5 = 2x + 1$ Substitute $2x - 5$ for $y$.
$-5 = 1$ ✗ Subtract $2x$ from each side.

▶ The equation $-5 = 1$ is never true. So, the system of linear equations has no solution.

### STUDY TIP

A linear system with no solution is called an *inconsistent system*.

## EXAMPLE 2 Solving a System: Infinitely Many Solutions

Solve the system of linear equations.

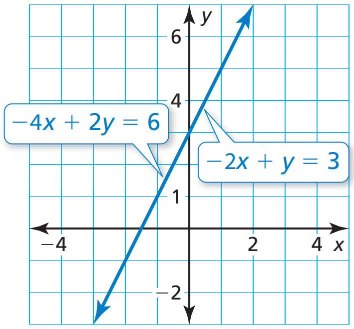
$-2x + y = 3$ Equation 1

$-4x + 2y = 6$ Equation 2

### SOLUTION

**Method 1** Solve by graphing.

Graph each equation.




The lines have the same slope and the same *y*-intercept. So, the lines are the same. Because the lines are the same, all points on the line are solutions of both equations.

▶ So, the system of linear equations has infinitely many solutions.

**Method 2** Solve by elimination.

**Step 1** Multiply Equation 1 by −2.

| | | | |
|---|---|---|---|
| $-2x + y = 3$ | **Multiply by −2.** → | $4x - 2y = -6$ | Revised Equation 1 |
| $-4x + 2y = 6$ | | $-4x + 2y = 6$ | Equation 2 |

**Step 2** Add the equations.

$4x - 2y = -6$ Revised Equation 1

$-4x + 2y = 6$ Equation 2

$0 = 0$ Add the equations.

**STUDY TIP**

A linear system with infinitely many solutions is called a *consistent dependent system*.

▶ The equation $0 = 0$ is always true. So, the solutions are all the points on the line $-2x + y = 3$. The system of linear equations has infinitely many solutions.

## Monitoring Progress

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**Solve the system of linear equations.**

**1.** $x + y = 3$
$2x + 2y = 6$

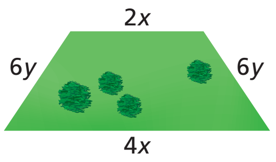
**2.** $y = -x + 3$
$2x + 2y = 4$

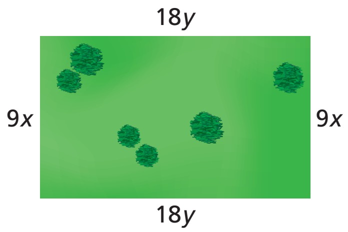
**3.** $x + y = 3$
$x + 2y = 4$

**4.** $y = -10x + 2$
$10x + y = 10$

## Solving Real-Life Problems

**EXAMPLE 3** **Modeling with Mathematics**




The perimeter of the trapezoidal piece of land is 48 kilometers. The perimeter of the rectangular piece of land is 144 kilometers. Write and solve a system of linear equations to find the values of *x* and *y*.

### SOLUTION

1. **Understand the Problem** You know the perimeter of each piece of land and the side lengths in terms of *x* or *y*. You are asked to write and solve a system of linear equations to find the values of *x* and *y*.
2. **Make a Plan** Use the figures and the definition of perimeter to write a system of linear equations that represents the problem. Then solve the system of linear equations.
3. **Solve the Problem**

**Perimeter of trapezoid**

$2x + 4x + 6y + 6y = 48$

$6x + 12y = 48$ Equation 1

**Perimeter of rectangle**

$9x + 9x + 18y + 18y = 144$

$18x + 36y = 144$ Equation 2

**System** $6x + 12y = 48$ Equation 1

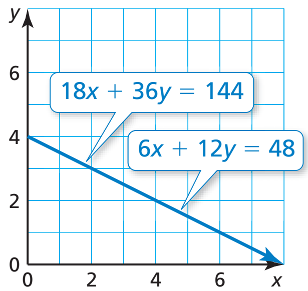
$18x + 36y = 144$ Equation 2

**Method 1** Solve by graphing.

Graph each equation.

The lines have the same slope and the same *y*-intercept. So, the lines are the same.

In this context, *x* and *y* must be positive. Because the lines are the same, all the points on the line in Quadrant I are solutions of both equations.




▶ So, the system of linear equations has infinitely many solutions.

**Method 2** Solve by elimination.

Multiply Equation 1 by −3 and add the equations.

$6x + 12y = 48$ **Multiply by −3.** $-18x - 36y = -144$ Revised Equation 1

$18x + 36y = 144$ $\quad 18x + 36y = 144$ Equation 2

$0 = 0$ Add the equations.

▶ The equation $0 = 0$ is always true. In this context, *x* and *y* must be positive. So, the solutions are all the points on the line $6x + 12y = 48$ in Quadrant I. The system of linear equations has infinitely many solutions.

4. **Look Back** Choose a few of the ordered pairs $(x, y)$ that are solutions of Equation 1. You should find that no matter which ordered pairs you choose, they will also be solutions of Equation 2. So, *infinitely many solutions* seems reasonable.

## Monitoring Progress

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5. **WHAT IF?** What happens to the solution in Example 3 when the perimeter of the trapezoidal piece of land is 96 kilometers? Explain.

Dynamic Solutions available at *BigIdeasMath.com*

## Vocabulary and Core Concept Check

1. **REASONING** Is it possible for a system of linear equations to have exactly two solutions? Explain.

2. **WRITING** Compare the graph of a system of linear equations that has infinitely many solutions and the graph of a system of linear equations that has no solution.

## Monitoring Progress and Modeling with Mathematics

**In Exercises 3–8, match the system of linear equations with its graph. Then determine whether the system has *one solution*, *no solution*, or *infinitely many solutions*.**

3. $-x + y = 1$
   $x - y = 1$

4. $2x - 2y = 4$
   $-x + y = -2$

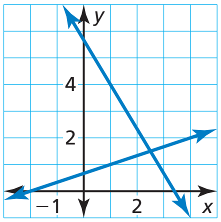
5. $2x + y = 4$
   $-4x - 2y = -8$

6. $x - y = 0$
   $5x - 2y = 6$

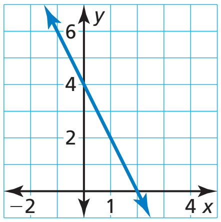
7. $-2x + 4y = 1$
   $3x - 6y = 9$

8. $5x + 3y = 17$
   $x - 3y = -2$

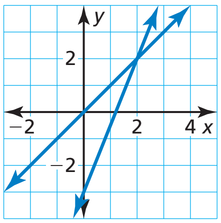
**A.**



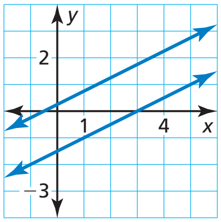

**B.**



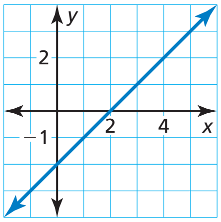

**C.**



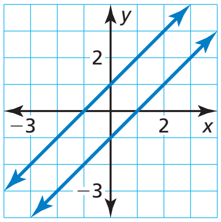

**D.**




**E.**




**F.**




**In Exercises 9–16, solve the system of linear equations.** *(See Examples 1 and 2.)*

9. $y = -2x - 4$
   $y = 2x - 4$

10. $y = -6x - 8$
    $y = -6x + 8$

11. $3x - y = 6$
    $-3x + y = -6$

12. $-x + 2y = 7$
    $x - 2y = 7$

13. $4x + 4y = -8$
    $-2x - 2y = 4$

14. $15x - 5y = -20$
    $-3x + y = 4$

15. $9x - 15y = 24$
    $6x - 10y = -16$

16. $3x - 2y = -5$
    $4x + 5y = 47$

**In Exercises 17–22, use only the slopes and *y*-intercepts of the graphs of the equations to determine whether the system of linear equations has *one solution*, *no solution*, or *infinitely many solutions*. Explain.**

17. $y = 7x + 13$
    $-21x + 3y = 39$

18. $y = -6x - 2$
    $12x + 2y = -6$

19. $4x + 3y = 27$
    $4x - 3y = -27$

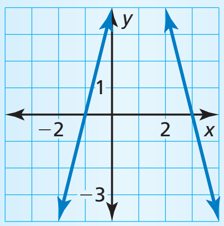
20. $-7x + 7y = 1$
    $2x - 2y = -18$

21. $-18x + 6y = 24$
    $3x - y = -2$

22. $2x - 2y = 16$
    $3x - 6y = 30$

**ERROR ANALYSIS In Exercises 23 and 24, describe and correct the error in solving the system of linear equations.**

23. 




$-4x + y = 4$
$4x + y = 12$

The lines do not intersect. So, the system has no solution.

24. $y = 3x - 8$
    $y = 3x - 12$

The lines have the same slope. So, the system has infinitely many solutions.

**25.** **MODELING WITH MATHEMATICS** A small bag of trail mix contains 3 cups of dried fruit and 4 cups of almonds. A large bag contains $4\frac{1}{2}$ cups of dried fruit and 6 cups of almonds. Write and solve a system of linear equations to find the price of 1 cup of dried fruit and 1 cup of almonds. *(See Example 3.)*


$9 $6

**26.** **MODELING WITH MATHEMATICS** In a canoe race, Team A is traveling 6 miles per hour and is 2 miles ahead of Team B. Team B is also traveling 6 miles per hour. The teams continue traveling at their current rates for the remainder of the race. Write a system of linear equations that represents this situation. Will Team B catch up to Team A? Explain.

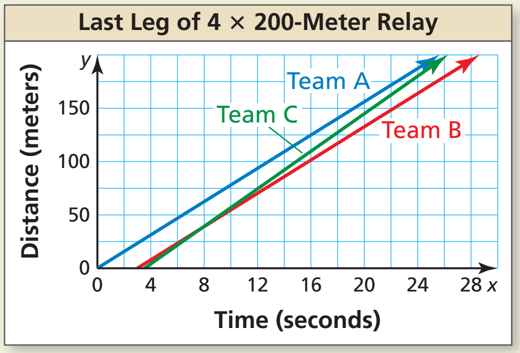
**27.** **PROBLEM SOLVING** A train travels from New York City to Washington, D.C., and then back to New York City. The table shows the number of tickets purchased for each leg of the trip. The cost per ticket is the same for each leg of the trip. Is there enough information to determine the cost of one coach ticket? Explain.

| Destination | Coach tickets | Business class tickets | Money collected (dollars) |
|---|---|---|---|
| Washington, D.C. | 150 | 80 | 22,860 |
| New York City | 170 | 100 | 27,280 |

**28.** **THOUGHT PROVOKING** Write a system of three linear equations in two variables so that any two of the equations have exactly one solution, but the entire system of equations has no solution.

**29.** **REASONING** In a system of linear equations, one equation has a slope of 2 and the other equation has a slope of $-\frac{1}{3}$. How many solutions does the system have? Explain.

**30.** **HOW DO YOU SEE IT?** The graph shows information about the last leg of a $4 \times 200$-meter relay for three relay teams. Team A's runner ran about 7.8 meters per second, Team B's runner ran about 7.8 meters per second, and Team C's runner ran about 8.8 meters per second.




**a.** Estimate the distance at which Team C's runner passed Team B's runner.

**b.** If the race was longer, could Team C's runner have passed Team A's runner? Explain.

**c.** If the race was longer, could Team B's runner have passed Team A's runner? Explain.

**31.** **ABSTRACT REASONING** Consider the system of linear equations $y = ax + 4$ and $y = bx - 2$, where $a$ and $b$ are real numbers. Determine whether each statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

**a.** The system has infinitely many solutions.

**b.** The system has no solution.

**c.** When $a < b$, the system has one solution.

**32.** **MAKING AN ARGUMENT** One admission to an ice skating rink costs $x$ dollars, and renting a pair of ice skates costs $y$ dollars. Your friend says she can determine the exact cost of one admission and one skate rental. Is your friend correct? Explain.


## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

**Solve the equation. Check your solutions.** *(Section 1.4)*

**33.** $|2x + 6| = |x|$

**34.** $|3x - 45| = |12x|$

**35.** $|x - 7| = |2x - 8|$

**36.** $|2x + 1| = |3x - 11|$