5.2 Solving Systems of Linear Equations by Substitution

Essential Question How can you use substitution to solve a system of linear equations?

EXPLORATION 1 Using Substitution to Solve Systems

Work with a partner. Solve each system of linear equations using two methods.

Method 1 Solve for x first.
Solve for x in one of the equations. Substitute the expression for x into the other equation to find y. Then substitute the value of y into one of the original equations to find x.

Method 2 Solve for y first.
Solve for y in one of the equations. Substitute the expression for y into the other equation to find x. Then substitute the value of x into one of the original equations to find y.

Is the solution the same using both methods? Explain which method you would prefer to use for each system.

a. $x + y = -7$
$-5x + y = 5$

b. $x - 6y = -11$
$3x + 2y = 7$

c. $4x + y = -1$
$3x - 5y = -18$

EXPLORATION 2 Writing and Solving a System of Equations

Work with a partner.

a. Write a random ordered pair with integer coordinates. One way to do this is to use a graphing calculator. The ordered pair generated at the right is $(-2, -3)$.

b. Write a system of linear equations that has your ordered pair as its solution.

c. Exchange systems with your partner and use one of the methods from Exploration 1 to solve the system. Explain your choice of method.

Choose two random integers between −5 and 5.

```
randInt(-5,5,2)
                {-2 -3}
```

ATTENDING TO PRECISION
To be proficient in math, you need to communicate precisely with others.

Communicate Your Answer

3. How can you use substitution to solve a system of linear equations?

4. Use one of the methods from Exploration 1 to solve each system of linear equations. Explain your choice of method. Check your solutions.

a. $x + 2y = -7$
$2x - y = -9$

b. $x - 2y = -6$
$2x + y = -2$

c. $-3x + 2y = -10$
$-2x + y = -6$

d. $3x + 2y = 13$
$x - 3y = -3$

e. $3x - 2y = 9$
$-x - 3y = 8$

f. $3x - y = -6$
$4x + 5y = 11$

5.2 Lesson

Core Vocabulary

Previous
system of linear equations
solution of a system of linear equations

What You Will Learn

- Solve systems of linear equations by substitution.
- Use systems of linear equations to solve real-life problems.

Solving Linear Systems by Substitution

Another way to solve a system of linear equations is to use substitution.

Core Concept

Solving a System of Linear Equations by Substitution

Step 1 Solve one of the equations for one of the variables.

Step 2 Substitute the expression from Step 1 into the other equation and solve for the other variable.

Step 3 Substitute the value from Step 2 into one of the original equations and solve.

EXAMPLE 1 Solving a System of Linear Equations by Substitution

Solve the system of linear equations by substitution.

$y = -2x - 9$ Equation 1

$6x - 5y = -19$ Equation 2

SOLUTION

Step 1 Equation 1 is already solved for y.

Step 2 Substitute $-2x - 9$ for y in Equation 2 and solve for x.

$6x - 5y = -19$ Equation 2

$6x - 5(-2x - 9) = -19$ Substitute $-2x - 9$ for y.

$6x + 10x + 45 = -19$ Distributive Property

$16x + 45 = -19$ Combine like terms.

$16x = -64$ Subtract 45 from each side.

$x = -4$ Divide each side by 16.

Step 3 Substitute -4 for x in Equation 1 and solve for y.

$y = -2x - 9$ Equation 1

$= -2(-4) - 9$ Substitute -4 for x.

$= 8 - 9$ Multiply.

$= -1$ Subtract.

The solution is $(-4, -1)$.

Check

Equation 1

$y = -2x - 9$

$-1 \stackrel{?}{=} -2(-4) - 9$

$-1 = -1$ ✓

Equation 2

$6x - 5y = -19$

$6(-4) - 5(-1) \stackrel{?}{=} -19$

$-19 = -19$ ✓

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Solve the system of linear equations by substitution. Check your solution.

1. $y = 3x + 14$
$y = -4x$

2. $3x + 2y = 0$
$y = \frac{1}{2}x - 1$

3. $x = 6y - 7$
$4x + y = -3$

EXAMPLE 2 Solving a System of Linear Equations by Substitution

Solve the system of linear equations by substitution.

$-x + y = 3$ Equation 1

$3x + y = -1$ Equation 2

ANOTHER WAY

You could also begin by solving for x in Equation 1, solving for y in Equation 2, or solving for x in Equation 2.

SOLUTION

Step 1 Solve for y in Equation 1.

$y = x + 3$ Revised Equation 1

Step 2 Substitute $x + 3$ for y in Equation 2 and solve for x.

$3x + y = -1$ Equation 2

$3x + (x + 3) = -1$ Substitute $x + 3$ for y.

$4x + 3 = -1$ Combine like terms.

$4x = -4$ Subtract 3 from each side.

$x = -1$ Divide each side by 4.

Step 3 Substitute -1 for x in Equation 1 and solve for y.

$-x + y = 3$ Equation 1

$-(-1) + y = 3$ Substitute -1 for x.

$y = 2$ Subtract 1 from each side.

The solution is $(-1, 2)$.

Algebraic Check

Equation 1

$-x + y = 3$

$-(-1) + 2 \stackrel{?}{=} 3$

$3 = 3$ ✓

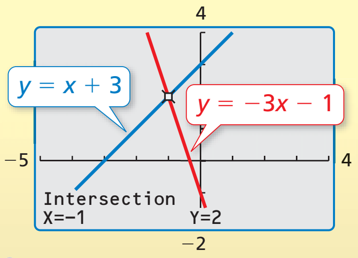
Equation 2

$3x + y = -1$

$3(-1) + 2 \stackrel{?}{=} -1$

$-1 = -1$ ✓

Graphical Check



Monitoring Progress

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Solve the system of linear equations by substitution. Check your solution.

4. $x + y = -2$
$-3x + y = 6$

5. $-x + y = -4$
$4x - y = 10$

6. $2x - y = -5$
$3x - y = 1$

7. $x - 2y = 7$
$3x - 2y = 3$

Solving Real-Life Problems

EXAMPLE 3 **Modeling with Mathematics**

A drama club earns \$1040 from a production. A total of 64 adult tickets and 132 student tickets are sold. An adult ticket costs twice as much as a student ticket. Write a system of linear equations that represents this situation. What is the price of each type of ticket?

SOLUTION

1. **Understand the Problem** You know the amount earned, the total numbers of adult and student tickets sold, and the relationship between the price of an adult ticket and the price of a student ticket. You are asked to write a system of linear equations that represents the situation and find the price of each type of ticket.
2. **Make a Plan** Use a verbal model to write a system of linear equations that represents the problem. Then solve the system of linear equations.
3. **Solve the Problem**

Words $64 \cdot \boxed{\text{Adult ticket price}} + 132 \cdot \boxed{\text{Student ticket price}} = 1040$

$\boxed{\text{Adult ticket price}} = 2 \cdot \boxed{\text{Student ticket price}}$

Variables Let x be the price (in dollars) of an adult ticket and let y be the price (in dollars) of a student ticket.

System $64x + 132y = 1040$ Equation 1

$x = 2y$ Equation 2

Step 1 Equation 2 is already solved for x.

Step 2 Substitute $2y$ for x in Equation 1 and solve for y.

$64x + 132y = 1040$ Equation 1

$64(2y) + 132y = 1040$ Substitute $2y$ for x.

$260y = 1040$ Simplify.

$y = 4$ Simplify.

Step 3 Substitute 4 for y in Equation 2 and solve for x.

$x = 2y$ Equation 2

$x = 2(4)$ Substitute 4 for y.

$x = 8$ Simplify.

▶ The solution is (8, 4). So, an adult ticket costs \$8 and a student ticket costs \$4.

4. **Look Back** To check that your solution is correct, substitute the values of x and y into both of the original equations and simplify.

$64(8) + 132(4) = 1040$ $8 = 2(4)$

$1040 = 1040$ ✓ $8 = 8$ ✓

STUDY TIP

You can use either of the original equations to solve for x. However, using Equation 2 requires fewer calculations.

Monitoring Progress

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8. There are a total of 64 students in a drama club and a yearbook club. The drama club has 10 more students than the yearbook club. Write a system of linear equations that represents this situation. How many students are in each club?

5.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** Describe how to solve a system of linear equations by substitution.

2. **NUMBER SENSE** When solving a system of linear equations by substitution, how do you decide which variable to solve for in Step 1?

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, tell which equation you would choose to solve for one of the variables. Explain.

3. $x + 4y = 30$
$x - 2y = 0$

4. $3x - y = 0$
$2x + y = -10$

5. $5x + 3y = 11$
$5x - y = 5$

6. $3x - 2y = 19$
$x + y = 8$

7. $x - y = -3$
$4x + 3y = -5$

8. $3x + 5y = 25$
$x - 2y = -6$

In Exercises 9–16, solve the sytem of linear equations by substitution. Check your solution. *(See Examples 1 and 2.)*

9. $x = 17 - 4y$
$y = x - 2$

10. $6x - 9 = y$
$y = -3x$

11. $x = 16 - 4y$
$3x + 4y = 8$

12. $-5x + 3y = 51$
$y = 10x - 8$

13. $2x = 12$
$x - 5y = -29$

14. $2x - y = 23$
$x - 9 = -1$

15. $5x + 2y = 9$
$x + y = -3$

16. $11x - 7y = -14$
$x - 2y = -4$

17. **ERROR ANALYSIS** Describe and correct the error in solving for one of the variables in the linear system $8x + 2y = -12$ and $5x - y = 4$.

Step 1
$$\begin{aligned} 5x - y &= 4 \\ -y &= -5x + 4 \\ y &= 5x - 4 \end{aligned}$$

Step 2
$$\begin{aligned} 5x - (5x - 4) &= 4 \\ 5x - 5x + 4 &= 4 \\ 4 &= 4 \end{aligned}$$

18. **ERROR ANALYSIS** Describe and correct the error in solving for one of the variables in the linear system $4x + 2y = 6$ and $3x + y = 9$.

Step 1
$$\begin{aligned} 3x + y &= 9 \\ y &= 9 - 3x \end{aligned}$$

Step 2
$$\begin{aligned} 4x + 2(9 - 3x) &= 6 \\ 4x + 18 - 6x &= 6 \\ -2x &= -12 \\ x &= 6 \end{aligned}$$

Step 3
$$\begin{aligned} 3x + y &= 9 \\ 3x + 6 &= 9 \\ 3x &= 3 \\ x &= 1 \end{aligned}$$

19. **MODELING WITH MATHEMATICS** A farmer plants corn and wheat on a 180-acre farm. The farmer wants to plant three times as many acres of corn as wheat. Write a system of linear equations that represents this situation. How many acres of each crop should the farmer plant? *(See Example 3.)*

20. **MODELING WITH MATHEMATICS** A company that offers tubing trips down a river rents tubes for a person to use and "cooler" tubes to carry food and water. A group spends \$270 to rent a total of 15 tubes. Write a system of linear equations that represents this situation. How many of each type of tube does the group rent?

In Exercises 21–24, write a system of linear equations that has the ordered pair as its solution.

21. $(3, 5)$

22. $(-2, 8)$

23. $(-4, -12)$

24. $(15, -25)$

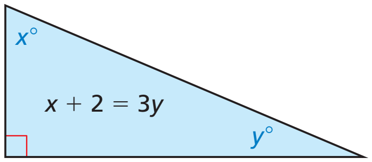
25. **PROBLEM SOLVING** A math test is worth 100 points and has 38 problems. Each problem is worth either 5 points or 2 points. How many problems of each point value are on the test?

26. **PROBLEM SOLVING** An investor owns shares of Stock A and Stock B. The investor owns a total of 200 shares with a total value of \$4000. How many shares of each stock does the investor own?

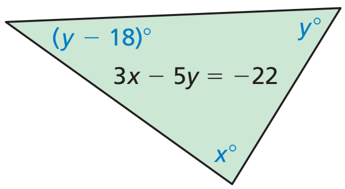
Stock	Price
A	\$9.50
B	\$27.00

MATHEMATICAL CONNECTIONS **In Exercises 27 and 28, (a) write an equation that represents the sum of the angle measures of the triangle and (b) use your equation and the equation shown to find the values of *x* and *y*.**

27.



28.



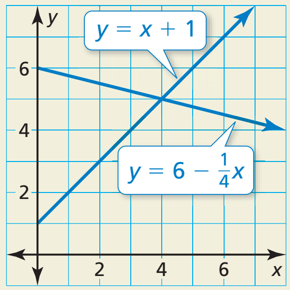
29. **REASONING** Find the values of a and b so that the solution of the linear system is $(-9, 1)$.

$$ax + by = -31 \quad \text{Equation 1}$$
$$ax - by = -41 \quad \text{Equation 2}$$

30. **MAKING AN ARGUMENT** Your friend says that given a linear system with an equation of a horizontal line and an equation of a vertical line, you cannot solve the system by substitution. Is your friend correct? Explain.

31. **OPEN-ENDED** Write a system of linear equations in which $(3, -5)$ is a solution of Equation 1 but not a solution of Equation 2, and $(-1, 7)$ is a solution of the system.

32. **HOW DO YOU SEE IT?** The graphs of two linear equations are shown.



a. At what point do the lines appear to intersect?

b. Could you solve a system of linear equations by substitution to check your answer in part (a)? Explain.

33. **REPEATED REASONING** A radio station plays a total of 272 pop, rock, and hip-hop songs during a day. The number of pop songs is 3 times the number of rock songs. The number of hip-hop songs is 32 more than the number of rock songs. How many of each type of song does the radio station play?

34. **THOUGHT PROVOKING** You have \$2.65 in coins. Write a system of equations that represents this situation. Use variables to represent the number of each type of coin.

35. **NUMBER SENSE** The sum of the digits of a two-digit number is 11. When the digits are reversed, the number increases by 27. Find the original number.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the sum or difference. *(Skills Review Handbook)*

36. $(x - 4) + (2x - 7)$

37. $(5y - 12) + (-5y - 1)$

38. $(t - 8) - (t + 15)$

39. $(6d + 2) - (3d - 3)$

40. $4(m + 2) + 3(6m - 4)$

41. $2(5v + 6) - 6(-9v + 2)$