4.7 Piecewise Functions

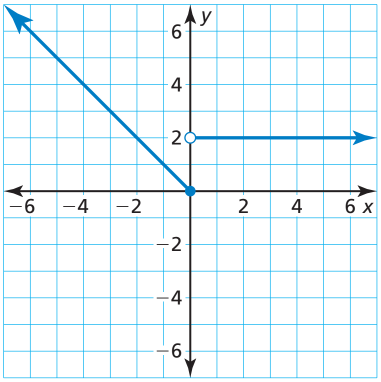
Essential Question How can you describe a function that is represented by more than one equation?

EXPLORATION 1 Writing Equations for a Function

Work with a partner.

CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to justify your conclusions and communicate them to others.



a. Does the graph represent y as a function of x? Justify your conclusion.

b. What is the value of the function when $x = 0$? How can you tell?

c. Write an equation that represents the values of the function when $x \leq 0$.

$f(x) = \underline{\hspace{3cm}}$, if $x \leq 0$

d. Write an equation that represents the values of the function when $x > 0$.

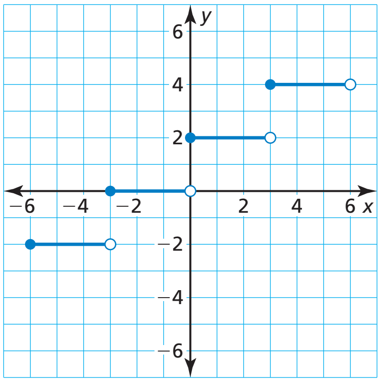
$f(x) = \underline{\hspace{3cm}}$, if $x > 0$

e. Combine the results of parts (c) and (d) to write a single description of the function.

$$f(x) = \begin{cases} \underline{\hspace{3cm}}, & \text{if } x \leq 0 \\ \underline{\hspace{3cm}}, & \text{if } x > 0 \end{cases}$$

EXPLORATION 2 Writing Equations for a Function

Work with a partner.



a. Does the graph represent y as a function of x? Justify your conclusion.

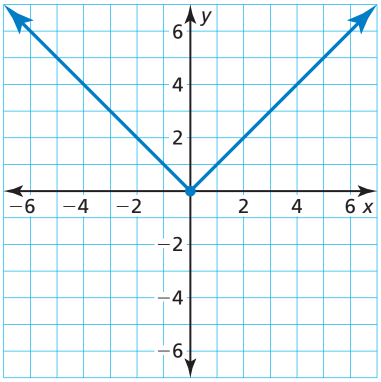
b. Describe the values of the function for the following intervals.

$$f(x) = \begin{cases} \underline{\hspace{3cm}}, & \text{if } -6 \leq x < -3 \\ \underline{\hspace{3cm}}, & \text{if } -3 \leq x < 0 \\ \underline{\hspace{3cm}}, & \text{if } 0 \leq x < 3 \\ \underline{\hspace{3cm}}, & \text{if } 3 \leq x < 6 \end{cases}$$

Communicate Your Answer

3. How can you describe a function that is represented by more than one equation?

4. Use two equations to describe the function represented by the graph.



4.7 Lesson

Core Vocabulary

piecewise function, *p. 218*
step function, *p. 220*

Previous
absolute value function
vertex form
vertex

What You Will Learn

- Evaluate piecewise functions.
- Graph and write piecewise functions.
- Graph and write step functions.
- Write absolute value functions.

Evaluating Piecewise Functions

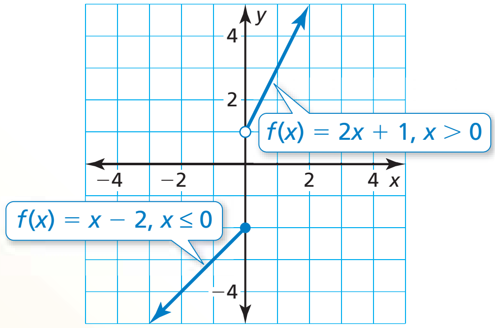
Core Concept

Piecewise Function

A **piecewise function** is a function defined by two or more equations. Each "piece" of the function applies to a different part of its domain. An example is shown below.

$$f(x) = \begin{cases} x - 2, & \text{if } x \le 0 \\ 2x + 1, & \text{if } x > 0 \end{cases}$$

- The expression $x - 2$ represents the value of f when x is less than or equal to 0.
- The expression $2x + 1$ represents the value of f when x is greater than 0.



EXAMPLE 1 Evaluating a Piecewise Function

Evaluate the function f above when (a) $x = 0$ and (b) $x = 4$.

SOLUTION

a. $f(x) = x - 2$ — Because $0 \le 0$, use the first equation.

$f(0) = 0 - 2$ — Substitute 0 for x.

$f(0) = -2$ — Simplify.

▶ The value of f is -2 when $x = 0$.

b. $f(x) = 2x + 1$ — Because $4 > 0$, use the second equation.

$f(4) = 2(4) + 1$ — Substitute 4 for x.

$f(4) = 9$ — Simplify.

▶ The value of f is 9 when $x = 4$.

Monitoring Progress

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Evaluate the function.

$$f(x) = \begin{cases} 3, & \text{if } x < -2 \\ x + 2, & \text{if } -2 \le x \le 5 \\ 4x, & \text{if } x > 5 \end{cases}$$

1. $f(-8)$ **2.** $f(-2)$

3. $f(0)$ **4.** $f(3)$

5. $f(5)$ **6.** $f(10)$

Graphing and Writing Piecewise Functions

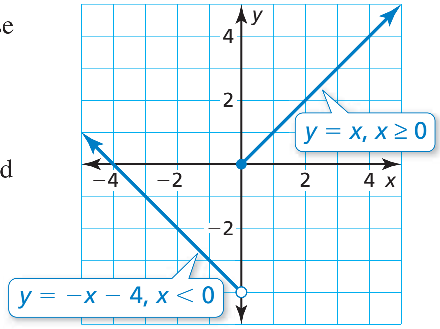
EXAMPLE 2 Graphing a Piecewise Function

Graph $y = \begin{cases} -x - 4, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$. Describe the domain and range.

SOLUTION

Step 1 Graph $y = -x - 4$ for $x < 0$. Because x is not equal to 0, use an open circle at $(0, -4)$.

Step 2 Graph $y = x$ for $x \geq 0$. Because x is greater than or equal to 0, use a closed circle at $(0, 0)$.



▶ The domain is all real numbers. The range is $y > -4$.

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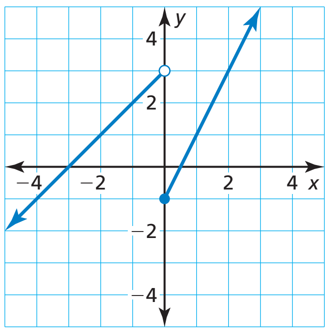
Graph the function. Describe the domain and range.

7. $y = \begin{cases} x + 1, & \text{if } x \leq 0 \\ -x, & \text{if } x > 0 \end{cases}$

8. $y = \begin{cases} x - 2, & \text{if } x < 0 \\ 4x, & \text{if } x \geq 0 \end{cases}$

EXAMPLE 3 Writing a Piecewise Function

Write a piecewise function for the graph.



SOLUTION

Each "piece" of the function is linear.

Left Piece When $x < 0$, the graph is the line given by $y = x + 3$.

Right Piece When $x \geq 0$, the graph is the line given by $y = 2x - 1$.

▶ So, a piecewise function for the graph is

$$f(x) = \begin{cases} x + 3, & \text{if } x < 0 \\ 2x - 1, & \text{if } x \geq 0 \end{cases}.$$

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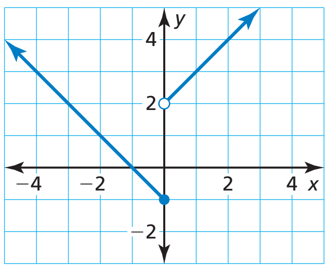
Write a piecewise function for the graph.

9.



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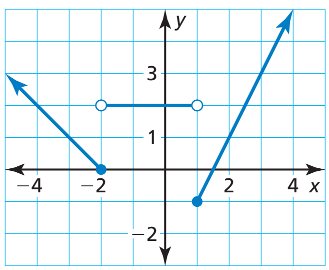
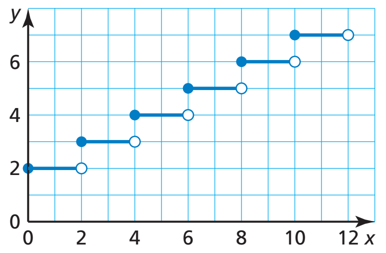


Graphing and Writing Step Functions

A **step function** is a piecewise function defined by a constant value over each part of its domain. The graph of a step function consists of a series of line segments.

STUDY TIP

The graph of a step function looks like a staircase.



$$f(x) = \begin{cases} 2, & \text{if } 0 \le x < 2 \\ 3, & \text{if } 2 \le x < 4 \\ 4, & \text{if } 4 \le x < 6 \\ 5, & \text{if } 6 \le x < 8 \\ 6, & \text{if } 8 \le x < 10 \\ 7, & \text{if } 10 \le x < 12 \end{cases}$$

EXAMPLE 4 Graphing and Writing a Step Function

You rent a karaoke machine for 5 days. The rental company charges \$50 for the first day and \$25 for each additional day. Write and graph a step function that represents the relationship between the number x of days and the total cost y (in dollars) of renting the karaoke machine.

SOLUTION

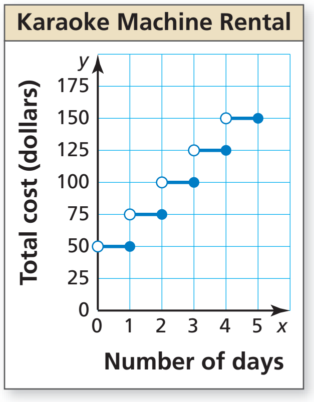
Step 1 Use a table to organize the information.

Number of days	Total cost (dollars)
$0 < x \le 1$	50
$1 < x \le 2$	75
$2 < x \le 3$	100
$3 < x \le 4$	125
$4 < x \le 5$	150

Step 2 Write the step function.

$$f(x) = \begin{cases} 50, & \text{if } 0 < x \le 1 \\ 75, & \text{if } 1 < x \le 2 \\ 100, & \text{if } 2 < x \le 3 \\ 125, & \text{if } 3 < x \le 4 \\ 150, & \text{if } 4 < x \le 5 \end{cases}$$

Step 3 Graph the step function.



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11. A landscaper rents a wood chipper for 4 days. The rental company charges \$100 for the first day and \$50 for each additional day. Write and graph a step function that represents the relationship between the number x of days and the total cost y (in dollars) of renting the chipper.

Writing Absolute Value Functions

The absolute value function $f(x) = |x|$ can be written as a piecewise function.

$$f(x) = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$$

Similarly, the vertex form of an absolute value function $g(x) = a|x - h| + k$ can be written as a piecewise function.

$$g(x) = \begin{cases} a[-(x - h)] + k, & \text{if } x - h < 0 \\ a(x - h) + k, & \text{if } x - h \geq 0 \end{cases}$$

REMEMBER

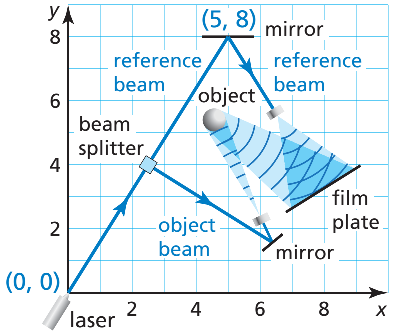
The vertex form of an absolute value function is $g(x) = a|x - h| + k$, where $a \neq 0$. The vertex of the graph of g is (h, k).

EXAMPLE 5 Writing an Absolute Value Function

In holography, light from a laser beam is split into two beams, a reference beam and an object beam. Light from the object beam reflects off an object and is recombined with the reference beam to form images on film that can be used to create three-dimensional images.

a. Write an absolute value function that represents the path of the reference beam.

b. Write the function in part (a) as a piecewise function.



SOLUTION

a. The vertex of the path of the reference beam is (5, 8). So, the function has the form $g(x) = a|x - 5| + 8$. Substitute the coordinates of the point (0, 0) into the equation and solve for a.

$g(x) = a|x - 5| + 8$ — Vertex form of the function

$0 = a|0 - 5| + 8$ — Substitute 0 for x and 0 for $g(x)$.

$-1.6 = a$ — Solve for a.

▶ So, the function $g(x) = -1.6|x - 5| + 8$ represents the path of the reference beam.

b. Write $g(x) = -1.6|x - 5| + 8$ as a piecewise function.

$$g(x) = \begin{cases} -1.6[-(x - 5)] + 8, & \text{if } x - 5 < 0 \\ -1.6(x - 5) + 8, & \text{if } x - 5 \geq 0 \end{cases}$$

Simplify each expression and solve the inequalities.

▶ So, a piecewise function for $g(x) = -1.6|x - 5| + 8$ is

$$g(x) = \begin{cases} 1.6x, & \text{if } x < 5 \\ -1.6x + 16, & \text{if } x \geq 5 \end{cases}.$$

STUDY TIP

Recall that the graph of an absolute value function is symmetric about the line $x = h$. So, it makes sense that the piecewise definition "splits" the function at $x = 5$.

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12. WHAT IF? The reference beam originates at (3, 0) and reflects off a mirror at (5, 4).

a. Write an absolute value function that represents the path of the reference beam.

b. Write the function in part (a) as a piecewise function.

Vocabulary and Core Concept Check

1. **VOCABULARY** Compare piecewise functions and step functions.

2. **WRITING** Use a graph to explain why you can write the absolute value function $y = |x|$ as a piecewise function.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–12, evaluate the function. *(See Example 1.)*

$$f(x) = \begin{cases} 5x - 1, & \text{if } x < -2 \\ x + 3, & \text{if } x \geq -2 \end{cases}$$

$$g(x) = \begin{cases} -x + 4, & \text{if } x \leq -1 \\ 3, & \text{if } -1 < x < 2 \\ 2x - 5, & \text{if } x \geq 2 \end{cases}$$

3. $f(-3)$
4. $f(-2)$
5. $f(0)$
6. $f(5)$
7. $g(-4)$
8. $g(-1)$
9. $g(0)$
10. $g(1)$
11. $g(2)$
12. $g(5)$

13. **MODELING WITH MATHEMATICS** On a trip, the total distance (in miles) you travel in x hours is represented by the piecewise function

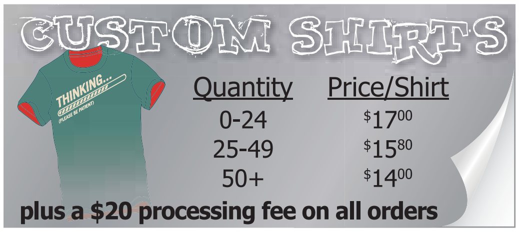
$$d(x) = \begin{cases} 55x, & \text{if } 0 \leq x \leq 2 \\ 65x - 20, & \text{if } 2 < x \leq 5 \end{cases}.$$

How far do you travel in 4 hours?

14. **MODELING WITH MATHEMATICS** The total cost (in dollars) of ordering x custom shirts is represented by the piecewise function

$$c(x) = \begin{cases} 17x + 20, & \text{if } 0 \leq x < 25 \\ 15.80x + 20, & \text{if } 25 \leq x < 50 \\ 14x + 20, & \text{if } x \geq 50 \end{cases}.$$

Determine the total cost of ordering 26 shirts.



In Exercises 15–20, graph the function. Describe the domain and range. *(See Example 2.)*

15. $y = \begin{cases} -x, & \text{if } x < 2 \\ x - 6, & \text{if } x \geq 2 \end{cases}$

16. $y = \begin{cases} 2x, & \text{if } x \leq -3 \\ -2x, & \text{if } x > -3 \end{cases}$

17. $y = \begin{cases} -3x - 2, & \text{if } x \leq -1 \\ x + 2, & \text{if } x > -1 \end{cases}$

18. $y = \begin{cases} x + 8, & \text{if } x < 4 \\ 4x - 4, & \text{if } x \geq 4 \end{cases}$

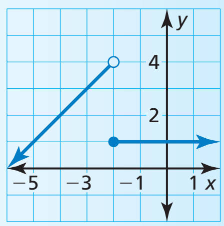
19. $y = \begin{cases} 1, & \text{if } x < -3 \\ x - 1, & \text{if } -3 \leq x \leq 3 \\ -2x + 4, & \text{if } x > 3 \end{cases}$

20. $y = \begin{cases} 2x + 1, & \text{if } x \leq -1 \\ -x + 2, & \text{if } -1 < x < 2 \\ -3, & \text{if } x \geq 2 \end{cases}$

21. **ERROR ANALYSIS** Describe and correct the error in finding $f(5)$ when $f(x) = \begin{cases} 2x - 3, & \text{if } x < 5 \\ x + 8, & \text{if } x \geq 5 \end{cases}$.

$$f(5) = 2(5) - 3$$
$$= 7$$

22. **ERROR ANALYSIS** Describe and correct the error in graphing $y = \begin{cases} x + 6, & \text{if } x \leq -2 \\ 1, & \text{if } x > -2 \end{cases}$.



In Exercises 23–30, write a piecewise function for the graph. *(See Example 3.)*

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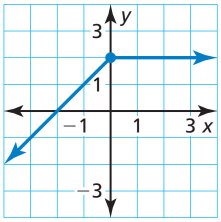

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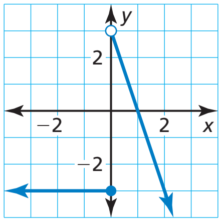

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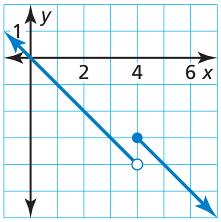

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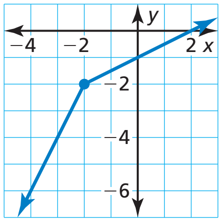

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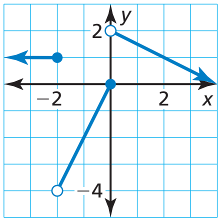

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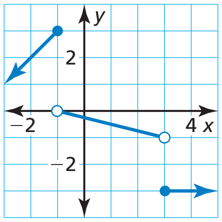

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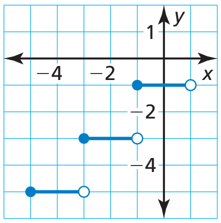

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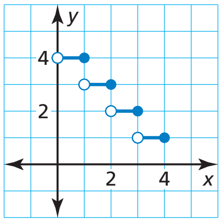


In Exercises 31–34, graph the step function. Describe the domain and range.

31. $f(x) = \begin{cases} 3, & \text{if } 0 \le x < 2 \\ 4, & \text{if } 2 \le x < 4 \\ 5, & \text{if } 4 \le x < 6 \\ 6, & \text{if } 6 \le x < 8 \end{cases}$

32. $f(x) = \begin{cases} -4, & \text{if } 1 < x \le 2 \\ -6, & \text{if } 2 < x \le 3 \\ -8, & \text{if } 3 < x \le 4 \\ -10, & \text{if } 4 < x \le 5 \end{cases}$

33. $f(x) = \begin{cases} 9, & \text{if } 1 < x \le 2 \\ 6, & \text{if } 2 < x \le 4 \\ 5, & \text{if } 4 < x \le 9 \\ 1, & \text{if } 9 < x \le 12 \end{cases}$

34. $f(x) = \begin{cases} -2, & \text{if } -6 \le x < -5 \\ -1, & \text{if } -5 \le x < -3 \\ 0, & \text{if } -3 \le x < -2 \\ 1, & \text{if } -2 \le x < 0 \end{cases}$

35. MODELING WITH MATHEMATICS The cost to join an intramural sports league is \$180 per team and includes the first five team members. For each additional team member, there is a \$30 fee. You plan to have nine people on your team. Write and graph a step function that represents the relationship between the number p of people on your team and the total cost of joining the league. *(See Example 4.)*

36. MODELING WITH MATHEMATICS The rates for a parking garage are shown. Write and graph a step function that represents the relationship between the number x of hours a car is parked in the garage and the total cost of parking in the garage for 1 day.

In Exercises 37–46, write the absolute value function as a piecewise function.

37. $y = |x| + 1$

38. $y = |x| - 3$

39. $y = |x - 2|$

40. $y = |x + 5|$

41. $y = 2|x + 3|$

42. $y = 4|x - 1|$

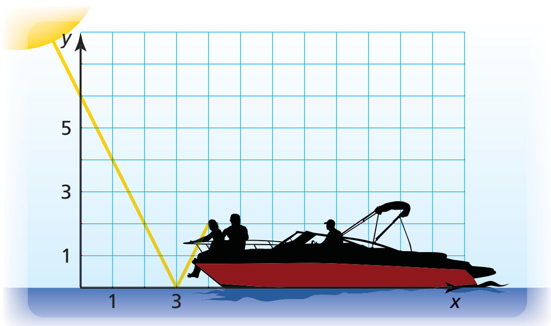
43. $y = -5|x - 8|$

44. $y = -3|x + 6|$

45. $y = -|x - 3| + 2$

46. $y = 7|x + 1| - 5$

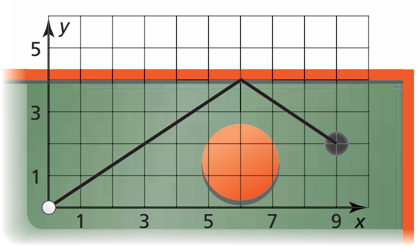
47. MODELING WITH MATHEMATICS You are sitting on a boat on a lake. You can get a sunburn from the sunlight that hits you directly and also from the sunlight that reflects off the water. *(See Example 5.)*



a. Write an absolute value function that represents the path of the sunlight that reflects off the water.

b. Write the function in part (a) as a piecewise function.

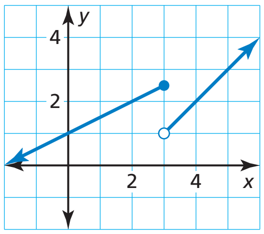
48. **MODELING WITH MATHEMATICS** You are trying to make a hole in one on the miniature golf green.



a. Write an absolute value function that represents the path of the golf ball.

b. Write the function in part (a) as a piecewise function.

49. **REASONING** The piecewise function f consists of two linear "pieces." The graph of f is shown.



a. What is the value of $f(-10)$?

b. What is the value of $f(8)$?

50. **CRITICAL THINKING** Describe how the graph of each piecewise function changes when $<$ is replaced with $\leq$ and $\geq$ is replaced with $>$. Do the domain and range change? Explain.

a. $f(x) = \begin{cases} x + 2, & \text{if } x < 2 \\ -x - 1, & \text{if } x \geq 2 \end{cases}$

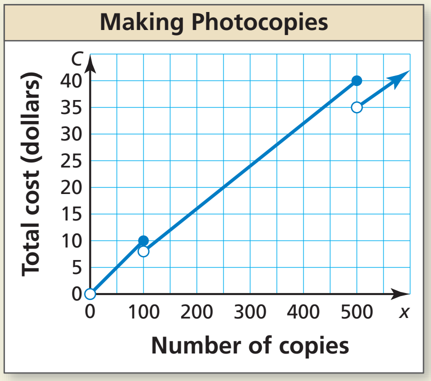
b. $f(x) = \begin{cases} \frac{1}{2}x + \frac{3}{2}, & \text{if } x < 1 \\ -x + 3, & \text{if } x \geq 1 \end{cases}$

51. **USING STRUCTURE** Graph

$$y = \begin{cases} -x + 2, & \text{if } x \leq -2 \\ \lvert x \rvert, & \text{if } x > -2 \end{cases}.$$

Describe the domain and range.

52. **HOW DO YOU SEE IT?** The graph shows the total cost C of making x photocopies at a copy shop.



a. Does it cost more money to make 100 photocopies or 101 photocopies? Explain.

b. You have \$40 to make photocopies. Can you buy more than 500 photocopies? Explain.

53. **USING STRUCTURE** The output y of the *greatest integer function* is the greatest integer less than or equal to the input value x. This function is written as $f(x) = [\![x]\!]$. Graph the function for $-4 \leq x < 4$. Is it a piecewise function? a step function? Explain.

54. **THOUGHT PROVOKING** Explain why

$$y = \begin{cases} 2x - 2, & \text{if } x \leq 3 \\ -3, & \text{if } x \geq 3 \end{cases}$$

does not represent a function. How can you redefine y so that it does represent a function?

55. **MAKING AN ARGUMENT** During a 9-hour snowstorm, it snows at a rate of 1 inch per hour for the first 2 hours, 2 inches per hour for the next 6 hours, and 1 inch per hour for the final hour.

a. Write and graph a piecewise function that represents the depth of the snow during the snowstorm.

b. Your friend says 12 inches of snow accumulated during the storm. Is your friend correct? Explain.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Write the sentence as an inequality. Graph the inequality. *(Section 2.5)*

56. A number r is greater than -12 and no more than 13.

57. A number t is less than or equal to 4 or no less than 18.

Graph f and h. Describe the transformations from the graph of f to the graph of h. *(Section 3.6)*

58. $f(x) = x$; $h(x) = 4x + 3$

59. $f(x) = x$; $h(x) = -x - 8$

60. $f(x) = x$; $h(x) = -\frac{1}{2}x + 5$